

Algebraic Sums for Deformation of Constructive Solids

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Abstract

This paper deals with an interactive sculpting technique based on local deformations of a constructive solid by a set of arbitrary points that are assumed to belong to the resultant surface. Representation of a solid by a real continuous function of three variables is used. The theory of R-functions is applied for set-theoretic operations on solids. In contrast to the existing methods based on space mapping, we construct a displacement function that interpolates values of the defining function in given control points. Then an algebraic sum (difference) of the defining function and the displacement function describes the deformed solid. Blobby deformation, deformation with a volume spline, and deformation controlled by a parametric curve are discussed. Blobby deformation models pinching and pricking that result in small bumps. To define more global deformation we use the volume spline function derived for scattered points and possessing minimal energy property. This approach has been applied to reconstruct solids from given surface points. Control of deformation with a parametric curve allows to produce sweep-like shapes placed on initial surfaces.

1 Deformations and algebraic sums

The sculpting metaphor of 3D shape modeling is intuitive and natural one for designers. Existing deformation methods are based on space mappings controlled by numerical parameters [3, 23], by point lattices [22, 9] and arbitrary control points [6, 12, 7]. Free-form deformations proposed by Sederberg and Parry [22] and extended in [9,12] can be effectively applied to polygonal and parametric surfaces. On the other hand, the inverse mapping for implicit surfaces requires time-consuming subdivisions or an iterative search. The general deformation techniques [6, 7] providing forward and inverse mapping better suit to implicit surfaces. Most of the above-mentioned deformation methods are really global to provide a series of small bumps defined by arbitrary points. Although the method of Borrel and Rappoport [7] has been designed for localized space mapping, it can lead to non intuitive results when bounding spheres of several control points intersect.

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The problem stated in our work is to control local deformations of a solid by a set of arbitrary points and curves that are assumed to belong to the surface of the resultant solid. Note that initial position of a point on the surface is not required. We would like to support creating a new feature by addition of just one control point. Pinching and pricking that result in small bumps can be dealt in this way. If we assume more smooth transition of the resultant surface between control points, more global deformation can be obtained.

We use representation of a solid by a continuous real function of three variables $f(x,y,z) \geq 0$. We call it a *defining function* of a solid. A zero set of such a function is usually called an implicit surface. We intend to apply algebraic operations (sum and difference) to implement deformations of functionally represented solids.

Algebraic operations on defining functions have been already applied to describe solids with implicit surfaces and geometric operations on them. Descriptions of implicit surfaces by blobby model [4], metaballs [14], soft objects [26] and distance functions [5, 13] are essentially based on algebraic sums of defining functions. It provides a possibility to deform an object by adding new primitives to its skeleton. Deformations of distance surfaces in collisions by algebraic difference of defining field functions have been proposed by Gascuel [11]. However, to construct an object of this kind one "must choose and place overlapping primitives with great skill in a most non intuitive way" [27].

A blend surface can be described in terms of algebraic operations on defining functions [25]. These operations are applied in practice to solid primitives but not to constructive solids (see, for example, [8]). The theory of R-functions [19] provides means of function representation of solids constructed by the standard (non-regularized) set operations (see [21] for a survey). Shapiro [21] applies algebraic difference to construct a real function defining a regular solids required in constructive solid geometry (CSG). Blending, offsetting and metamorphosis operations have been defined in [17,18,15] by algebraic sums applied to R-function based exact descriptions of constructive solids. Pasko and Savchenko [16] describe stochastic deformation of a solid using an algebraic sum with so-called "solid noise" function. To model these geometric operations we use a high-level geometric language [1] supporting mixed algebraic and set operations on defining functions.

As this brief overview shows, algebraic operations on defining functions give a promising tool for complex transformations of geometric solids. The general idea of our approach to deformations is to construct a displacement function that interpolates values of the defining function in given control points. Then algebraic

difference between the defining function and the displacement function describes the deformed solid. Next Section describes our approach and gives several examples. Future research directions are given in Section 3. A spline based on the Green's function is presented in Appendix.

2 Deformations with algebraic sums

Let $f_1(x,y,z)$ be a defining function of an initial solid G_1 and $\{p_i\}=\{(x_i,y_i,z_i)\}$, $i=1,N$ be a set of N arbitrary points of Euclidean space E^3 . These control points define deformation of G_1 resulting in a solid G_2 . Control points are assumed to belong to the surface of G_2 . The problem is to construct a defining function $f_2(x,y,z)$ for G_2 on the base of f_1 and $\{p_i\}$. We propose to express this dependence as an algebraic sum:

$$f_2(x,y,z) = f_1(x,y,z) + d(x,y,z),$$

where $d(x,y,z) = -f(x,y,z)$ is a continuous real displacement function.

2.1 Blobby deformation

We have found the following approximate form of the displacement function to be suitable in practical applications:

$$d(x,y,z) = - \sum_{i=1}^N \frac{f_1(x_i,y_i,z_i)}{1+(r_i/a_i)^2}$$

where $r_i^2 = (x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2$. This function has a local extremum in every control point. That is why it produces blobby-like deformation of a solid. The parameter a_i is used to control width of blobs. Note that this method does not guarantee connectivity of the resultant solid. In the case of a disconnected blob, the parameter a_i can help to produce connection. Suggested form of the displacement function is not time consuming, but also is not the only one possible. Other Gaussian-like functions can be designed for specific applications.

The simplest example is deformation of a primitive solid defined by an analytically expressed function. Deformation of a ball is illustrated by Fig.1. Note that white markers in Fig.1a correspond to control points positioned inside the solid and red markers correspond to control points outside the solid. These control points define pinched and pricked blobs correspondingly. To apply the same type of deformation to a constructive solid, we need to define it by a single continuous real function. Fig.2 illustrates deformation of a cube with a cylindrical hole defined by the set-theoretic operations. We have applied following R-functions to construct the defining function of this solid:

$$\begin{aligned} f_C &= f_A + f_B + \sqrt{f_A^2 + f_B^2} && \text{for union of solids A and B} \\ f_C &= f_A + f_B - \sqrt{f_A^2 + f_B^2} && \text{for intersection of solids A and B.} \end{aligned}$$

Fig.3 illustrates a blobby deformation of a solid defined by the set-theoretic and blending operations. A single real function defining the initial solid was constructed by means of the modified R-functions [17] used for blending intersection of solids A and B:

$$f_C = f_A + f_B - \sqrt{f_A^2 + f_B^2} + \frac{b_0}{1 + (f_A/b_1)^2 + (f_B/b_2)^2}$$

The initial solid and 38 control points are shown in Fig 3a. Note, some control points are outside the solid (letters "E", "A" and part of "S") and other inside the solid (part of "S" and "Y" letter) but all of them are near the surface. The resulting blobby deformation is shown in Fig.3b. Outside points result in pinching effect and inside points result in pricking effect. One can notice local nature of deformations that is practically impossible to achieve by another existing methods.

If one needs to introduce an elliptic area of influence for each blob and orientation of its local coordinate system, the following form of the displacement function can be suitable:

$$d(x,y,z) = - \sum_{i=1}^N d_i(x,y,z)$$

$$\text{where } d_i(x,y,z) = \frac{f_1(x_i,y_i,z_i) \cdot (1-r_i^2)^3}{1+r_i^2} \quad \text{if } r_i^2 < 1$$

$$\text{and } d_i(x,y,z) = 0 \quad \text{if } r_i^2 \geq 1$$

for $r_i^2 = (x' / a_i)^2 + (y' / a_i)^2 + (z' / a_i)^2$, where (x', y', z') are coordinates of a point in a local coordinate system of the i -th blob. This type of Gaussian-like function has been proposed by Wyvill et al. [28] for parametric surface design. Fig.3c illustrates modeling of scratches by this displacement function with 15 control points.

2.2 Deformation with a volume spline

If a displacement function approximates defining function values in control points more smoothly (without local extreme points), more global deformation can be obtained. On the other hand, it can be located in a certain space area. We use the volume spline function derived for multidimensional scattered data on the base of 'thin plate' function [10, 2] and Green's function [24] (see Appendix). Fig.4 shows a constructive solid deformed with this volume spline. Deflated and inflated versions of the "rubber" object are defined by a few inside and outside control points. No local bumps have appeared and the deformation has quite global nature. Deformations can be localized easily by proper placing control points. Additional control points with predetermined zero spline values can help as well.

This approach has been applied also to reconstruct solids from given surface points and contours [20]. A solid being searched is

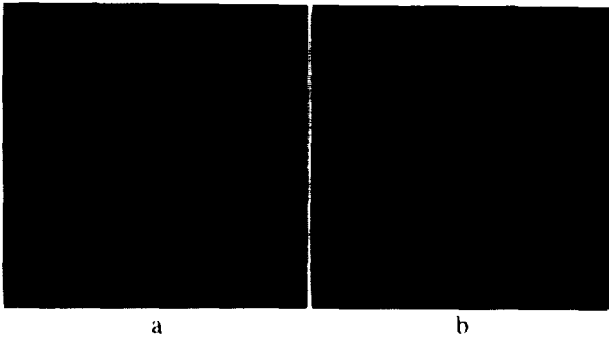


Figure 1. Deformation of a solid ball with five arbitrarily placed control points. The white markers correspond to the control points positioned outside the solid and the red markers correspond to the control points inside the solid.

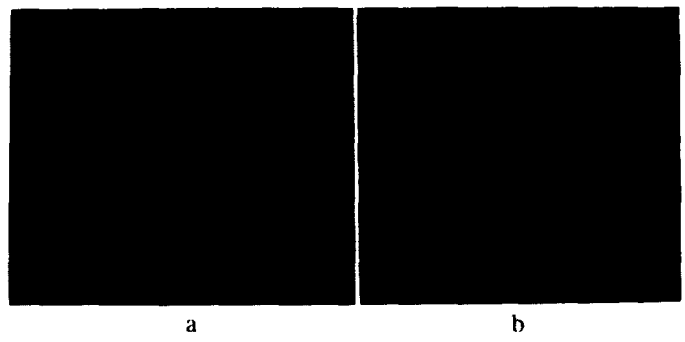


Figure 2. Deformation of a cube with a cylindrical hole defined by the set-theoretic operations using R-functions. Seven control points have been placed inside (four upper points) and outside the cube (three lower points).

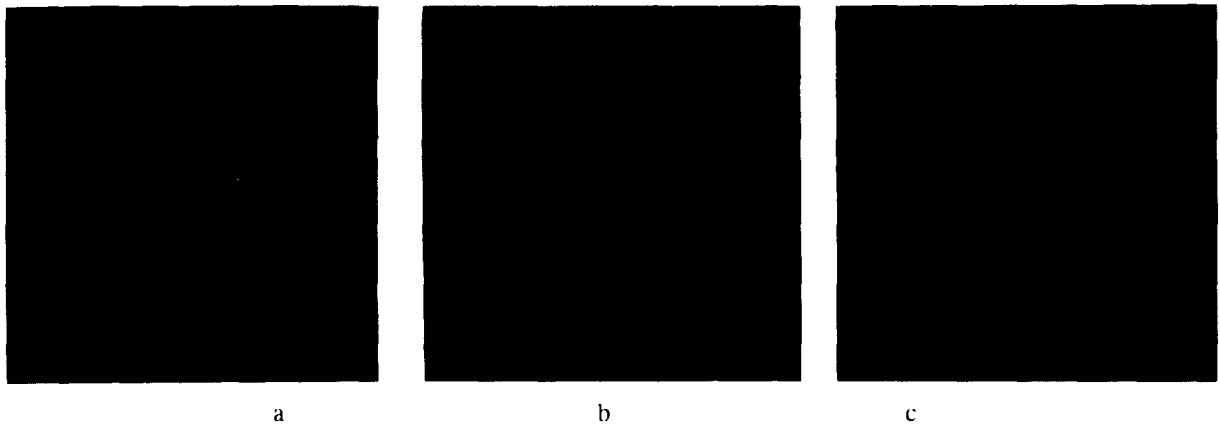


Figure 3. Blobby deformation of a solid defined by the set-theoretic and blending operations: a) initial solid and 38 control points; b) pinching and pricking effect of deformation; c) deformation by 15 blobs with elliptic area of influence provides scratching effect.



Figure 4. Deformation with a volume spline: centre - a "rubber" cat with control points, left - a deflated cat defined by eight control points (red markers), right - an inflated cat defined by six control points (white markers).

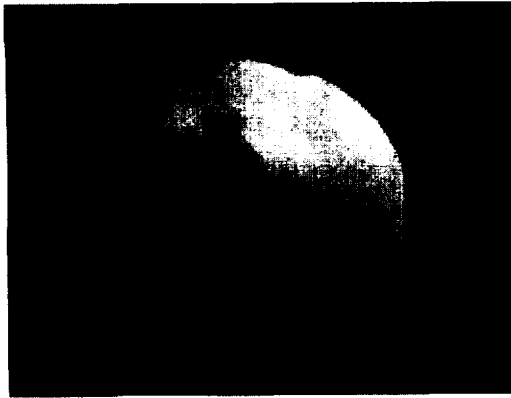


Figure 5. Reconstructed Mars satellite

represented as a “carrier” solid deformed using an algebraic sum with a volume spline. Reconstructed solid model of the Mars satellite (Fobos) is shown in Fig.5. A ball as a “carrier” solid was deformed by algebraic sum with the volume spline interpolating function values in 942 points randomly placed on the surface.

2.3 Deformation with a parametric curve

It might be interesting to control a deformation by a continuous curve rather than by a finite set of control points. Then the displacement function can be defined as

$$d(x,y,z) = - \int_0^1 f_1(x(t), y(t), z(t)) e^{-r^2} dt$$

where $r^2 = (x-x(t))^2 + (y-y(t))^2 + (z-z(t))^2$ and functions $x(t)$, $y(t)$, $z(t)$ define a control parametric curve. Fig.6 illustrates this type of deformation with two different procedures of integral calculation. An initial block is deformed under control of a parametric spiral curve. The deformed shape in Fig.6a is produced using Simpson's method of integral calculation. In the second case (Fig.6b) the integral was estimated as a maximal value of the integrated function. We have applied the iterative quadratic search method to find the maximum. Although the deformed shape in Fig.6b more strongly depends on the features of the initial object (for example near the edges), it took 2.5 times fewer number of function estimations to calculate this deformation. One could notice a sweep-like shape produced by this deformation technique.

3 Conclusion and future work

We have presented a new technique for deformation of geometric solids defined by real functions of three variables. This technique is applicable to solids constructed with set-theoretic operations and described using R-functions. The chosen types of displacement functions allow producing both subtle local bumps and more global

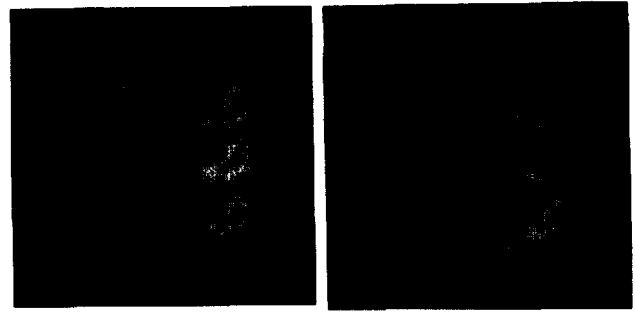


Figure 6. Deformation of a block with a control parametric spiral: a) numerical integration; b) an integral estimated as a maximal value of a function.

deformation. Only positions of arbitrary points, but not their displacements, are needed to control deformation. It is quite intuitive and simplifies interactive process very much.

Future work will focus on three areas. First, combination of space transformations and algebraic sums on the base of multidimensional mapping will be investigated. It can provide flexible control of global and local deformations in a uniform manner. The second area of interest is to develop deformation controlled by parametric surface patches. The third area involves providing user control of the deformation in virtual reality environment.

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Appendix. Volume spline based on Green's function

Data given:

- Scattered points $\{P_i = (x_1^i, x_2^i, \dots, x_n^i): i=1, \dots, N\}$ in E^n
- Values of a defining function $f_i = f(P_i)$

The problem is to find a smooth function $U(x)$ so that $U(P_i) = f_i$. The Green's function is used as a basis function:

$$G_{m,n}(x, P_i) = \begin{cases} \|x-P_i\|^{2m-n} \ln \|x-P_i\| & \text{if } n \text{ is even} \\ \|x-P_i\|^{2m-n} & \text{if } n \text{ is odd,} \end{cases}$$

where $\|x-P_i\| = (\sum_{j=1}^n (x_j - x_j^i)^2)^{1/2}$, $x = (x_1, x_2, \dots, x_n)$ is an arbitrary point of E^n .

The coefficient $m \geq 2$ defines an applied norm. In practice, $m=2$ can be used. For $m=2$ and $n=2,3$ the spline has the following form:

$$U(x) = \sum_{i=1}^{N+k} \lambda_i g_i(x, P_i),$$

where $g_i(x, P_i) = G_{m_i}(x, P_i)$, $i=1, \dots, N$,
 $g_{N+1}(x, P_i) = 1$,
 $g_{N+i_j}(x, P_i) = x_j$, $j=1, \dots, k-1$,
 $k = (n+m-1)! / (n!(m-1)!)$.

The spline coefficients λ_i are calculated using the system of $(N+k)$ linear equations:

$$\mathbf{A} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \dots \\ \lambda_N \\ \lambda_{N+1} \\ \lambda_{N+2} \\ \dots \\ \lambda_{N+k} \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ \dots \\ f_N \\ 0 \\ 0 \\ \dots \\ 0 \end{pmatrix}$$

$$\begin{aligned} A_{ij} &= g_j(P_i, P_j), & i \leq N+k, j \leq N, i \neq j; \\ A_{ii} &= 0; & i < N; \\ A_{ij} &= g_j(P_i, P_j), & i \leq N, N < j \leq N+k; \\ A_{ij} &= 0, & i > N, j > N. \end{aligned}$$

The system is solved by Householder's method. After defining the coefficients the spline $U(x)$ can be restored. The choice of this spline has been caused by the following reasons:

- it was derived especially for the case of scattered points;
- minimal energy property;
- C^k continuity with $k < 2m - n$.

References

1. Adzhiev V.D., Beynon W.M., Pasko A.A. Interactive geometric modelling based on R-functions: an agent-oriented approach, Set-theoretic Solid Modeling: Techniques and Applications, CSG 94 Conference Proceedings, Information Geometers, Winchester, UK, 1994, pp 253-272.
2. Alfeld P. Scattered data interpolation in three or more variables, Mathematical Methods in Computer Aided Geometric Design, T.Lyche and L.Schumaker (Ed.), Academic Press, 1989, pp.1-33.
3. Barr A.H. Global and local deformations of solid primitives, Computer Graphics, vol.18, No.3, 1984, pp.21-30.
4. Blinn J.F. A generalization of algebraic surface drawing, ACM Transactions on Graphics, vol.1, No.3, 1982, pp.235-256.
5. Bloomenthal J., Wyvill B. Interactive techniques for implicit modeling, Computer Graphics, vol.24, No.2, 1990, pp 109-116.
6. Borrel P., Bechmann D. Deformation of N-dimensional objects, International Journal of Computational Geometry and Applications, vol.1, No.4, 1991, pp.427-453.
7. Borrel P., Rappoport A. Simple constrained deformations for geometric modeling and interactive design, ACM Transactions on Graphics, vol.13, No.2, 1994, pp.137-155.

8. Bowyer A. SVLIS Set-Theoretic Kernel Modeller. Introduction and User Manual, Information Geometers, 1994, 116 p.
9. Coquillart S. Extended free-form deformation: a sculpting tool for 3D geometric modeling, Computer Graphics, vol.24, No.4, 1990, pp.187-196.
10. Duchon J. Splines minimizing rotation-invariant semi-norms in Sobolev spaces, Constructive Theory of Functions of Several Variables, A.Dodd and B.Eckmann (Ed.), Springer-Verlag, 1977, pp.85-100.
11. Gascuel M.P. An implicit formulation for precise contact modeling between flexible solids, SIGGRAPH'93, Computer Graphics Proceedings, Annual Conference Series, 1993, pp.313-320.
12. Hsu W.M., Hughes G.F., Kaufman H. Direct manipulation of free-form deformations, Computer Graphics, vol.26, No.2, 1992, pp.177-184.
13. Larcombe M.H.E. Generalised distance functions and plastic modelling over an armature, Computer-aided Surface Geometry and Design, The Mathematics of Surfaces IV, Ed. A.Bowyer, Clarendon Press, Oxford, 1994, pp.281-298.
14. Nishimura H., Hirai M., Kawai T., Kawata T., Shirakawa I., Omura K. Object modeling by distribution function and a method of image generation, Journal of papers given at the Electronics Communication Conference'85, J68-D (4) (in Japanese).
15. Pasko A., Savchenko V., Adzhiev V., Sourin A. Multidimensional geometric modeling and visualization based on the function representation of objects, TR 93-1-008, The University of Aizu, Japan, 1993, 47 p.
16. Pasko A.A., Savchenko V.V. Solid noise in the constructive solid geometry, EDUGRAPHICS'93 and COMPUGRAPHICS'93, Combined Proceedings, Ed. H.P.Santo, ACM, Portugal, 1993, pp.351-357.
17. Pasko A.A., Savchenko V.V. Blending operations for the functionally based constructive geometry, Set Theoretic Solid Modeling: Techniques and Applications, CSG 94 Conference Proceedings, Information Geometers, Winchester, UK, 1994, pp.151-161.
18. Pasko A.A., Savchenko V.V. Offsetting operations for the functionally based constructive geometry, Proceedings of 6th International Conference on Engineering Computer Graphics and Descriptive Geometry, Tokyo, Japan, 1994, vol.1, pp.201-204.
19. Rvachev V.L. Theory of R-functions and Some Applications, Naukova Dumka, Kiev, 1987 (in Russian)
20. Savchenko V.V., Pasko A.A., Okunev O.G., Kunii T.L. Function representation of solids reconstructed from scattered surface points and contours, TR 94-1-032, The University of Aizu, Japan, 1994, 13 p.
21. Shapiro V. Real functions for representation of rigid solids, Computer Aided Geometric Design, vol.11, No.2, 1994, pp.153-175.
22. Sederberg T.W., Parry S.R. Free-form deformation of solid geometric models, Computer Graphics, vol.20, No.4, 1986,

pp.151-160.

23. Sclaroff S., Pentland A. Generalized implicit functions for computer graphics, *Computer Graphics*, vol.25, No.4, 1991, pp.247-250.

24. Vasilenko V.A. *Spline-functions: Theory, Algorithms and Programs*, Nauka Publishers, Novosibirsk, 1983 (in Russian).

25. Woodwark J.R. Blends in geometric modeling, *The Mathematics of Surfaces II*, Ed. R.R.Martin, Oxford University Press, 1987, pp.255-297.

26. Wyvill G., McPheeters C., Wyvill B. Data structure for soft objects, *The Visual Computer*, vol.2, No.4, 1986, pp.227-234.

27. Wyvill G., McRobie D. Local and global control of Cao En surfaces, *Communicating with Virtual Worlds, CGI'93 Proceedings*, Eds. N.Magenat Thalmann and D.Thalmann, Springer-Verlag, 1993, pp.216-227.

28. Wyvill G., McRobie D., Haig C. Free form modelling with history, *Proceedings of International Workshop 'Shape Modeling: Parallelism, Interactivity and Applications'*, University of Aizu, Japan, 1994, pp.69-75.