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Bounded Blending for Function-based Shape Modeling

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Abstract

We propose new analytical formulations of bounded blending operations for the function-based constructive shape modeling. The blending set operations are defined using R-functions and displacement functions with the localized area of influence. The shape and location of the blend is defined by control points on the surfaces of two solids or by an additional arbitrary bounding solid also defined by a real-valued function. The proposed blending using a bounding solid can be applied to a single selected edge, a vertex, or another blend. We introduce new types of blends such as a multiple blend with the disconnected bounding solid and a partial edge blend. We show that the proposed operations can replace pure set-theoretic operations in the solid model without rebuilding the entire construction tree data structure. The proposed blending is shown to have versatile applications in interactive design. Influence of all parameters on the blend shape and location is illustrated.

Keywords: Geometric modeling, shape, blending, implicit surfaces, R-functions

1. Introduction

Blending operations in shape modeling generate smooth transitions between two or several surfaces. Blending is considered a natural property of implicit surfaces [2], where the basic operation is an algebraic sum (or difference) between skeleton-based scalar fields. Blending operations are also used in computer-aided design for modeling fillets and chamfers, but have different nature.[13,17] These operations are usually smooth versions of set-theoretic operations on solids (intersection, union, and difference), which approximate exact results of these operations by rounding sharp edges and vertices.

Detailed discussions of blending techniques can be found in [2,13,17]. The major requirements to blending operations summarized from the above papers are the following:

- Tangency of the blend surface with the initial surfaces: blending operation should not create additional sharp edges;
- Automatic clipping of unwanted parts of the blending surface: only behavior of the blending surface inside of the blending area is the matter of interest;
- Exact analytical or procedural definitions of blends instead of approximations;
- C^1 continuity of the blending function everywhere in the domain;
- Blending definition of the basic set-theoretic operations: standard set-theoretic operations result in sharp edges, which need to be smoothed by blending.
- Support of added and subtracted material blends;
- Intuitive control of the blend shape and position: parameters of blending should have clear geometric interpretation;
- Blended objects should be legal arguments of all operations applicable to unblended objects;
- Blend on blend operation should be supported including complex vertices where three or more surfaces are blended;

- Blending of a single selected edge has to be supported;
- Support of constant-radius blending and variable-radius blending.

Note that some requirements are overlapping or even contradictory to each other. Several methods have been proposed for blending implicit surfaces and for blending versions of set-theoretic operations (see the discussion in the next section). However, the need existed in the continuous analytical description of set-theoretic operations with controllable blend shapes.

In [10], an analytical definition of blending set-theoretic operations was proposed. For the solids exactly represented by continuous real functions, the authors tried to satisfy most of the above requirements. The theory of R-functions and their extensions were used to define blending operations. The major disadvantage of this approach is that the generated blends are global and no way was proposed for their localization. In this paper, we extend the approach of [10] by introducing bounded blending operations defined using R-functions and displacement functions with the localized area of influence. The shape and location of the blend is defined by control points on the surfaces of two solids or by an additional bounding solid. The proposed blending using a bounding solid can be applied to a single selected edge or vertex. We describe several experiments and introduce new types of blends such as a multiple blend with the disconnected bounding solid and a partial edge blend. Set-theoretic operations on blends and blends on blends (i.e., recursive blends) are also supported. Therefore, the proposed operations can replace pure set-theoretic operations in the construction of a solid without rebuilding the entire construction tree data structure. The proposed blending is shown to have versatile applications in interactive design. Influence of all parameters on the blend shape and location is illustrated.

2. Other works

Natural blending is considered one of the advantages of implicit surface modeling.[2] The blending effect appears when applying an algebraic sum (or difference) to two or several skeleton-based scalar fields. In the case of blending of two disconnected surfaces, a single blend surface can be generated. Controlled blending has been tackled in implicit surface modeling research in the context of skeletal primitives [4,18], convolution surfaces [1], or multiple intersecting implicit surfaces.[5] The addressed problems are control of the overall shape of blend [3,18] and blend on blend [5], preventing unwanted blending in cases of contact between different surfaces or a folding skeleton curve. [1,4]

Note that the major operations in implicit surface modeling (algebraic sum and difference) do not describe set-theoretic union or subtraction, which are key operations in solid modeling. Let us consider the works particularly oriented to blending versions of set-theoretic operations on solids with implicit surfaces [7,10,11]. The blending method of [11] is based on the approximations of min/max functions used for set-theoretic operations. The proposed blending has global character in the sense that the entire initial surfaces are replaced by the blending surface. This formulation provides only added material blend for union and subtracted material blend for intersection. Constant-radius blends [13] are defined using the concept of a rolling sphere and the offset operation over constructive solids. The work[7] provides analytical definitions for blending surfaces. However, added or subtracted material blends require additional set-theoretic operations for the appropriate truncation of the unwanted parts of the blend solid. An analytical definition of blending set-theoretic operations was proposed in [10]. This definition covered added and subtracted material blends as well as symmetric and asymmetric blending. The main problem with this approach is that the blends are global in nature and cannot be localized using the blend parameters. The local blends that control how far along a surface the blend adheres were proposed in [6, 14].

A method for controlling the blend shape using a free-form two-dimensional curve is proposed in [3]. Although this method provides means for interactive design of complex shape transitions between implicit surfaces, its possibilities to localize the blend are quite limited. For example, the specified blend is applied to the entire intersection edge of two surfaces and can not be limited to a part of this edge. In this work, we extend the approach of [10] to provide means for localization of blending effects for R-functions based set-theoretic operations.

3. Function representation and set-theoretic operations

The function representation (FRep) was introduced in [8] as a generalization of traditional implicit surfaces, Constructive Solid Geometry (CSG), sweeping, and other solid models.[12] In FRep, an entire 3D object is represented by a single continuous real function of three variables as $F(x,y,z) \geq 0$. A point belongs to the object if the function is positive at the point. The function is zero on the entire surface (called traditionally an *implicit surface*) of the object and is negative at any point outside the object. In a FRep modeling system, an object is represented by a tree data structure reflecting the logical structure of the object construction, where leaves are arbitrary primitives and nodes are arbitrary operations described by real functions. Function evaluation procedures traverse the tree and evaluate the function value at any given point.

Many modeling operations have been formulated, which are closed on the representation, i.e., generate as a result another continuous function defining the transformed FRep object. The major operations bridging the gap between implicit surfaces (used as FRep primitives) and CSG are set-theoretic operations. R-functions [8,15,16] provide exact analytical definitions of set-theoretic operations with C^1 and higher-level continuity of the resulting function. An object resulting from set-theoretic operations has the defining function as follows:

$f_3 = f_1 \vee_\alpha f_2$ for the union;
$f_3 = f_1 \wedge_\alpha f_2$ for the intersection;
$f_3 = f_1 \setminus_\alpha f_2$ for the subtraction,

where f_1 and f_2 are defining functions of initial objects and $\vee_\alpha, \wedge_\alpha, \setminus_\alpha$ are signs of R-functions. One of the possible analytical descriptions of the R-functions is as follows:

$$f_1 \vee_\alpha f_2 = \frac{1}{1+\alpha}(f_1 + f_2 + \sqrt{f_1^2 + f_2^2 - 2\alpha f_1 f_2})$$
$$f_1 \wedge_\alpha f_2 = \frac{1}{1+\alpha}(f_1 + f_2 - \sqrt{f_1^2 + f_2^2 - 2\alpha f_1 f_2})$$

where $\alpha = \alpha(f_1, f_2)$ is an arbitrary continuous function satisfying the following conditions:

$$-1 < \alpha(f_1, f_2) < 1,$$
$$\alpha(f_1, f_2) = \alpha(f_2, f_1) = \alpha(-f_1, f_2) = \alpha(f_1, -f_2)$$

The expression for the subtraction operation is

$$f_1 \setminus_\alpha f_2 = f_1 \wedge_\alpha (-f_2)$$

Note that with this definition of the subtraction, the resulting object includes its boundary.
If α=1, the above functions become

$$f_1 \wedge_1 f_2 = \min(f_1, f_2)$$
$$f_1 \vee_1 f_2 = \max(f_1, f_2)$$

These R-functions are very convenient for calculations but have C^1 discontinuity when $f_1 = f_2$. If α =0, the above general formulation takes the most useful in practice form:

$$f_1 \vee_0 f_2 = f_1 + f_2 + \sqrt{f_1^2 + f_2^2}$$
$$f_1 \wedge_0 f_2 = f_1 + f_2 - \sqrt{f_1^2 + f_2^2}$$

These functions have C^1 discontinuity only in points where both arguments are equal to zero. We use this class of R-functions in the following sections to introduce bounded blending versions of set-theoretic operations.

4. Blending based on R-functions

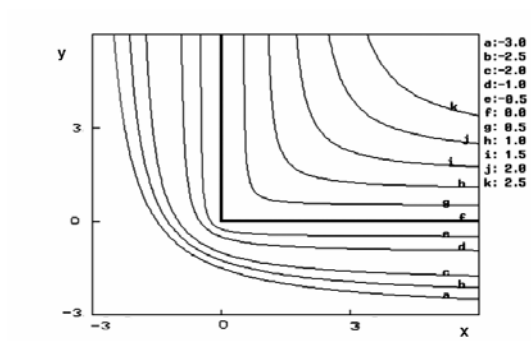


Figure 1. Contour lines of the R-function defining the intersection of halfplanes $x \geq 0$ and $y \geq 0$.

In [10], an analytical definition of blending set-theoretic operations on solids with implicit surfaces was proposed. This work was motivated by observing the properties of R-functions. In Figure 1, contour lines of the function $F(x,y)$ defining the intersection of halfplanes $x \geq 0$ and $y \geq 0$ are shown. Using the theory of R-functions, the result of the intersection can be described as:

$$F(x, y) = x + y - \sqrt{x^2 + y^2}$$

This function has a curve with a sharp vertex (bold line in Figure 1) as a zero value iso-contour $F(x,y)=0$. Other contour lines are smooth in the entire domain. This property brings the idea that some displacement of the exact R-function can result in the blending effect between the lines $x = 0$ and $y = 0$.

The following definition of the blending set-theoretic operation was proposed in [10]:

$$F_b(f_1, f_2) = R(f_1, f_2) + disp_b(f_1, f_2), \qquad (1)$$

where $R(f_1, f_2)$ is an R-function corresponding to the type of the operation, the arguments of the operation $f_1(X)$ and $f_2(X)$ are defining functions of two initial solids, and $disp(f_1, f_2)$ is a Gaussian-type displacement function. The following expression for the displacement function was used:

$$disp_b(f_1, f_2) = \frac{a_0}{1 + \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2}, \qquad (2)$$

where a_0, a_1, $a_2 \in$ R are parameters controlling the shape of the blend. The proposed definition is suitable for blending union, intersection, and difference, allows for generating added and subtracted material, symmetric and asymmetric blends. As it was shown above, the intersection operation can be described by an R-function as

$$R_{\text{int}}(f_1, f_2) = f_1 + f_2 - \sqrt{f_1^2 + f_2^2} \qquad (3)$$

with the corresponding blending intersection described as

$$F_{b\,\text{int}} = f_1 + f_2 - \sqrt{f_1^2 + f_2^2} + \frac{a_0}{1 + \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2}$$

The proposed displacement function does not get zero value anywhere in the space. This is the reason of the main disadvantage of this definition - the blend has global character and cannot be localized using its parameters. In this paper, we propose to use different displacement functions, which allow for localization of the blend using control points or an additional bounding solid.

5. Bounded blending

In this section, we propose new formulations for blending operations with the blend bounded by control points and by an additional bounding solid. The definition (Equation 1) of a blending operation is used as the basis with the modifications introduced only for the displacement function. First, we select a displacement function with the local area of influence (section 5.1), then, we construct a generalized distance function for bounding blends using control points (section 5.2) and bounding solids (section 5.3).

5.1 Displacement function

As the first step of defining bounded blending, we propose to select for the definition (Equation 1) a displacement function of one variable $disp_{bb}(r)$, which in contrast to the displacement function in [10] has a localized support area and takes zero value outside of this area. To specify the support area we use a generalized distance r to both initial surfaces, where r is constructed using defining functions

of the initial solids as it will be shown in the next section. New displacement function has to satisfy the following conditions:

1) $disp_{bb}(r) \geq 0$, $disp_{bb}(r)$ takes the maximal value for *r=0*, and the displacement is symmetric in respect to the initial defining functions;
2) $disp_{bb}(r) = 0, r \geq 1$ is a condition of the blend localization;
3) $\frac{\partial disp_{bb}}{\partial r} = 0, r = 1$ means the curve $disp_{bb}(r)$ tangentially approaches the horizontal axis at *r=1* and accordingly the blend tangentially approaches initial surfaces.

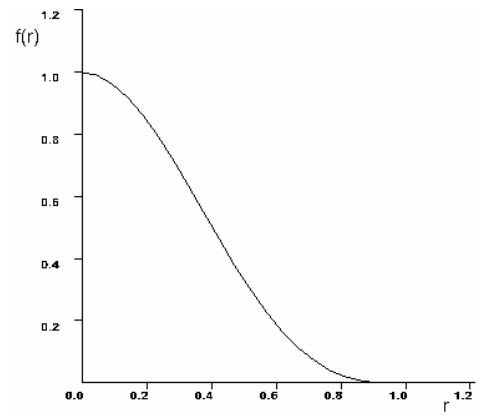


Figure 2. Displacement function for bounded blending.

A plot of the desired displacement function is shown in Figure 2. One can derive or find several analytical expressions for such function. In fact, this work has been done by many researchers for localizing individual components of skeletal implicit surfaces. [2] One of the possible expressions, which we will use in the rest of the paper, is

$$disp_{bb}(r) = \begin{cases} \frac{(1-r^2)^3}{1+r^2}, r < 1 \\ 0, r \geq 1 \end{cases} \qquad (4)$$

5.2 Bounding by control points

We discuss here the first and possibly the simplest method of the blend localization. The idea is to place one control point on the surface of each initial solid and to require that the blend exists only "between" these control points. The control points are supposed to be on or relatively close to the edge resulting from the pure set-theoretic operation and it should be intuitively obvious what area is designated for blending. To satisfy such requirement, the displacement function (Equation 4) has to take zero value at the control points with the maximal value on or near the edge. Let us suppose that the initial defining functions $f_1(X)$ and $f_2(X)$ have distance properties (continuously decreasing outside the respective solid). Then, the following generalized distance function can be proposed:

$$r^2(f_1, f_2) = \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2, \qquad (5)$$

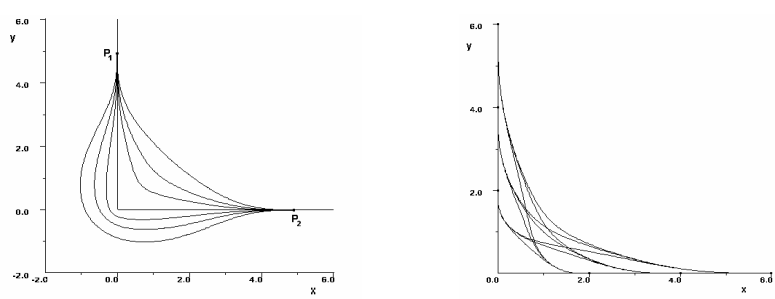
where $f_1(X)$ and $f_2(X)$ are defining functions of two solids as arguments of the blending operation; $a_1 = f_1(P_2)$, where P_2 is a control point placed on the surface $f_2(X) = 0$; $a_2 = f_2(P_1)$, where P_1 is a control point placed on the surface $f_1(X) = 0$. Note the following properties of this function:

1) $r^2(0,0) = 0$;
2) at the point P_1: $f_1(P_1) = 0, f_2(P_1) = a_2,$ and $r^2(f_1, f_2) = 1$;
3) at the point P_2: $f_1(P_2) = a_1, f_2(P_2) = 0,$ and $r^2(f_1, f_2) = 1$.

These properties mean that the displacement function $disp_{bb}(r)$ (Equation 4) takes maximal value at the intersection points of two surfaces, where $f_1(X) = 0$ and $f_2(X) = 0$, and becomes zero at control points P_1 and P_2. Using Equation 1 and applying the displacement function $disp_{bb}(r)$ defined by Equation 4 with r^2 defined by Equation 5, we get for the bounded blending operations with control points:

$$F_{bb}(f_1, f_2) = R(f_1, f_2) + a_0 disp_{bb}(r), \quad (6)$$

where a real numerical parameter a_0 is added to control the weight of the displacement function. Examples of such bounded blending operations are given in Figures 3-4. In Figure 3, we illustrate the bounded blending intersection of two halfplanes $f_1(x, y) \geq 0$ and $f_2(x, y) \geq 0$, where $f_1(x, y) = x$ and $f_2(x, y) = y$. The control point P_1 is placed on the line $x = 0$, the boundary of the first halfplane, and the control point P_2 is placed on the line $y = 0$, the boundary of the second halfplane. One can observe that the blend is located between the control points due to the displacement function definition. In Figure 3a, the positive values of the parameter a_0 correspond to the added material blends, $a_0 = 0$ keeps the result of the pure intersection and the negative values of a_0 correspond to subtracted material blends. If the parameter a_0 goes to infinity, the additional material completely fills the area $r \leq 1$; if the parameter a_0 goes to negative infinity, the solid $r \leq 1$ is subtracted from both initial solids. Moving control points along the initial boundaries allows for generating symmetric and asymmetric blends as it is illustrated by Figure 3b.



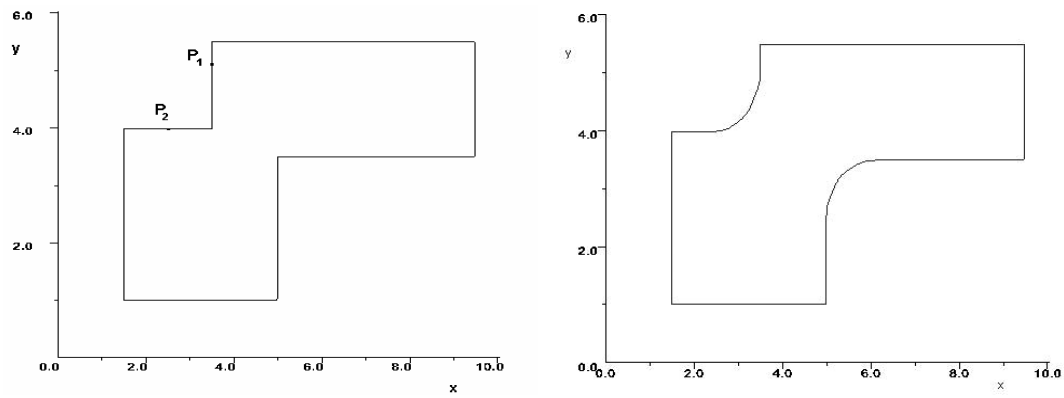
a b

Figure. 3 Bounded blending intersection of halfplanes $x \geq 0$ and $y \geq 0$ with control points P_1 and P_2:

a) added and subtracted material blends controlled by the parameter a_0;
b) symmetric and asymmetric blends with different positions of control points.

The bounded blending union operation of two rectangles with control points is illustrated by Figure 4. Two control points are placed on the corresponding boundaries of the rectangles (Figure 4a). The

expected bounded blend appears between the control points at the upper left part of the shape in Figure 4b. However, the rectangles are blended also in the lower right part of the shape, which can be unexpected or unwanted. The reason of this unwanted behavior is that control points are not directly used for bounding the blend, but their coordinates are used in the functional formulation (Equation 5), which works for any other pair of points with the same function values. This example unveils the global character of the blend with control points, which does not allow a designer to select a single vertex or an edge for blending. Another means of defining bounds are needed to provide truly local character of blending.



a b

Figure 4. Union of two rectangles and two control points (a); expected bounded blend between control points at the upper left part of the shape (b) and unwanted blend at the lower right part of the shape (b).

5.3 Bounding solid

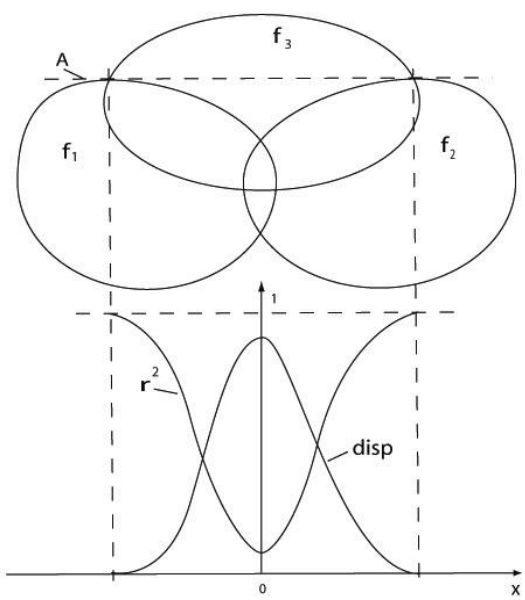


Figure 5. Components of the definition of the bounded blending union. Upper part: two solids to be blended (f_1 and f_2) and a bounding solid f_3. Lower part: behavior of the functions r^2 and *disp* in the cross-section A of the bounding solid.

The introduction of an additional bounding solid can bring real local character to the blend. It is required that the blending surface exists only inside the bounding solid, and only original surfaces exist outside the bounding solid. The upper part of Figure 5 shows two solids to be blended (defined by the functions f_1 and f_2) and a bounding solid (defined by the function f_3). To apply the bounded blending definition

$$F_{bb}(f_1, f_2, f_3) = R(f_1, f_2) + a_0 disp_{bb}(r) \quad (7)$$

with the displacement function (4), we have to provide the following interconnected properties (see lower part of Figure 5):

1) Zero value of the displacement function $disp_{bb}(r)$ outside the bounding solid, where the generalized distance r takes value 1; the displacement function tangentially approaching the horizontal axis provides tangency of blending surface and initial surfaces;
2) The maximal value of the displacement function $disp_{bb}(r)$ near the intersection points of two initial surfaces;
3) The generalized distance function r provides the algebraic distance measure to both initial surfaces; it takes zero value at the intersection points of two initial surfaces;
4) The generalized distance function r is modulated by the defining function f_3 of the bounding solid such that r takes value 1 on the boundary and outside the bounding solid; tangency of this function at the boundary of the bounding solid provides the required tangency of the displacement function.

The following definitions satisfy the above requirements:

$$disp_{bb}(r) = \begin{cases} \dfrac{(1-r^2)^3}{1+r^2}, r<1 \\ 0, r \geq 1 \end{cases} \quad \text{with } r^2 = \begin{cases} \dfrac{r_1^{\,2}}{r_1^{\,2} + r_2^{\,2}}, r_2 > 0 \\ 1, r_2 = 0 \end{cases}, \quad (8)$$

where

$$r_1^{\,2}(f_1, f_2) = \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2,$$

and

$$r_2^{\,2}(f_3) = \begin{cases} \left(\dfrac{f_3}{a_3}\right)^2, f_3 > 0 \\ 0, f_3 \leq 0 \end{cases},$$

with non-zero numerical parameters a_1 and a_2 controlling the blend symmetry, and a_3 allowing the user to interactively control the influence of the function f_3 on the overall shape of the blend. Note that the displacement function $disp_{bb}(r)$ tangentially approaches the zero value at the boundary of the bounding solid with $r = 1$ (see Figure 2), which provides the tangency of the blend to the initial surfaces at their intersection points with the bounding solid.

The above definitions (Equation 8) of the generalized distance r and of the displacement function $disp_{bb}(r)$ are not unique and can be changed, if it is necessary in particular applications. We explore the meaning of parameters, the properties and applications of the proposed bounded blending in the next section.

6. Blend properties and applications

In this section, we illustrate such properties of the proposed blending operations with bounding solids as their local character and intuitive control of blend shape and position. The analytical definition of blends allows for support of such unusual operations as multiple blending and partial edge blending.

6.1 Feature selection for blending

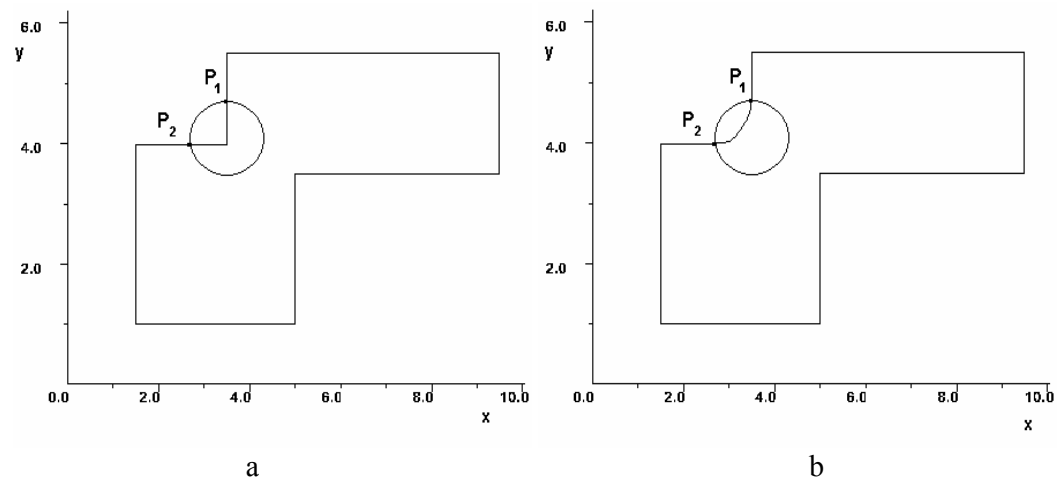


Figure 6. Blending union at a single vertex selected by the bounding disk.

The use of a bounding solid allows the user to select a single feature (vertex or edge) of the constructive solid for blending. The corresponding pure set-theoretic operation should be replaced by the bounded blending operation. In Figure 6, we present an example of union of two rectangular solids in 2D. A 2D disk is used as a solid bounding the blend in the area near a single vertex (Figure 6a). The pure union operation is replaced by the bounded blending union (Equation 7) and the result is shown in Figure 6b. The expected bounded blend appears inside the bounding disk in the upper left part of the shape in Figure 6b. However, in comparison with Figure 4b, no blending appears in the lower right part of the shape. This example illustrates the local character of the blending with a bounding solid.

6.2 Control of blend shape and position

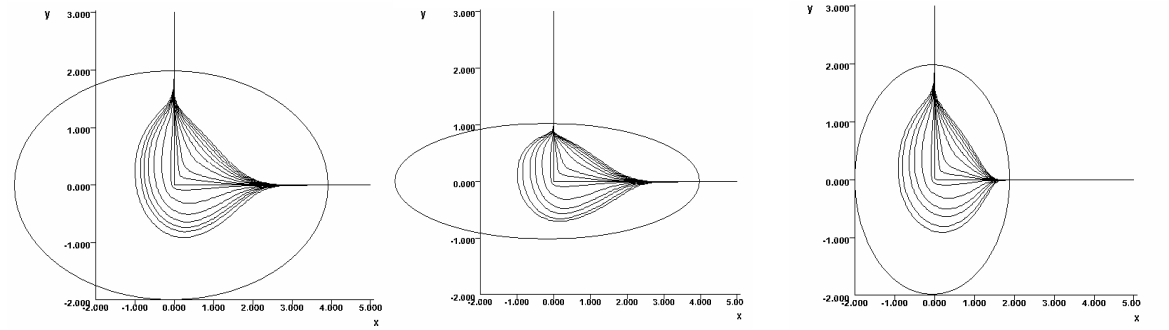
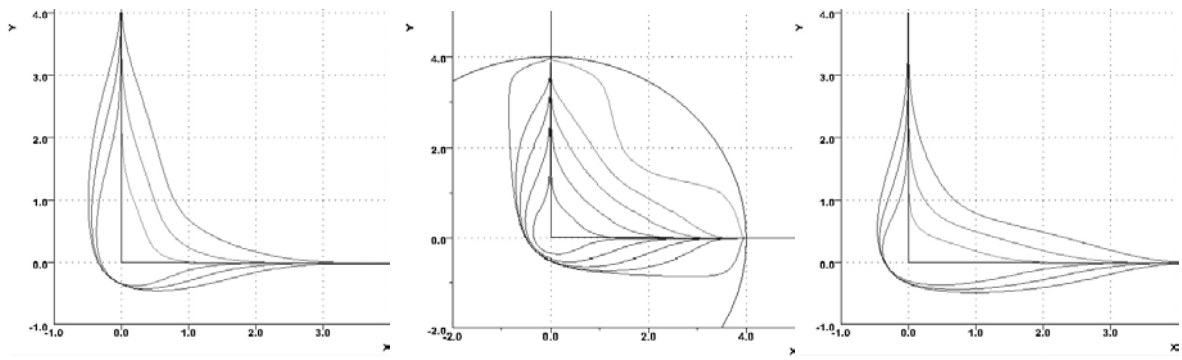


Figure 7. Shapes of 2D blends are controlled by changing parameters of the bounding ellipsoid.



$a_1 < a_2$ $\qquad$ $a_1 = a_2$ $\qquad$ $a_1 > a_2$

Figure 8. Influence of the blending parameters on the shape of blends with added material ($a_0 > 0$) and subtracted material ($a_0 < 0$) for the intersection of two half-planes $f_1 = x$ and $f_2 = y$ with the bounding disk $f_3 = 1-(x/4)^2-(y/4)^2$. The parameter a_3 is proportional to the distance of the blend curve to the bounding circle (boundary of the bounding disk): blends with higher a_3 values are smaller in size and located closer to the origin (bounding disk center), blends with smaller a_3 values are closer to the bounding circle (note the extreme cases of outer curves in the central image).

The shape and position of the blend is controlled by its parameters and by the position and shape of the bounding solid. A family of blends inside the bounding ellipse is shown in Figure 7. Subtracted material blends are described by Equation 7 with negative a_0 values, added material blends correspond to positive a_0 . Figure 7 also shows that the blend changes its shape following the changes of the bounding solid. Figure 8 illustrates influence of the blend parameters (Equations 7, 8) on the blend shape for the intersection of two half-planes $f_1 = x$ and $f_2 = y$ with the bounding disk $f_3 = 1-(x/4)^2-(y/4)^2$. The following properties of the blend can be mentioned:

• The absolute value of a_0 defines the total displacement of the blending surface from the two initial surfaces.

• $a_0 = 0$ means pure set-theoretic operation.

• A positive a_0 value yields added material blend and a negative a_0 value gives subtracted material blend.

• Values of $a_1 > 0$ and $a_2 > 0$ are proportional to the distance between the blending surface and the original surfaces defined by f_2 and f_1, respectively. Symmetric blends (Figure 8 center), and asymmetric blends (Figure 8 left and right) can be generated by changing these parameters.

• Parameter a_3 controls the influence of the bounding solid on the overall blend shape. It is proportional to the distance of the blend surface to the surface of the bounding solid. The outer curves in the central image of Figure 8 show the extreme cases with enough small a_3 values the blend shape to start approaching the bounding blend boundary.

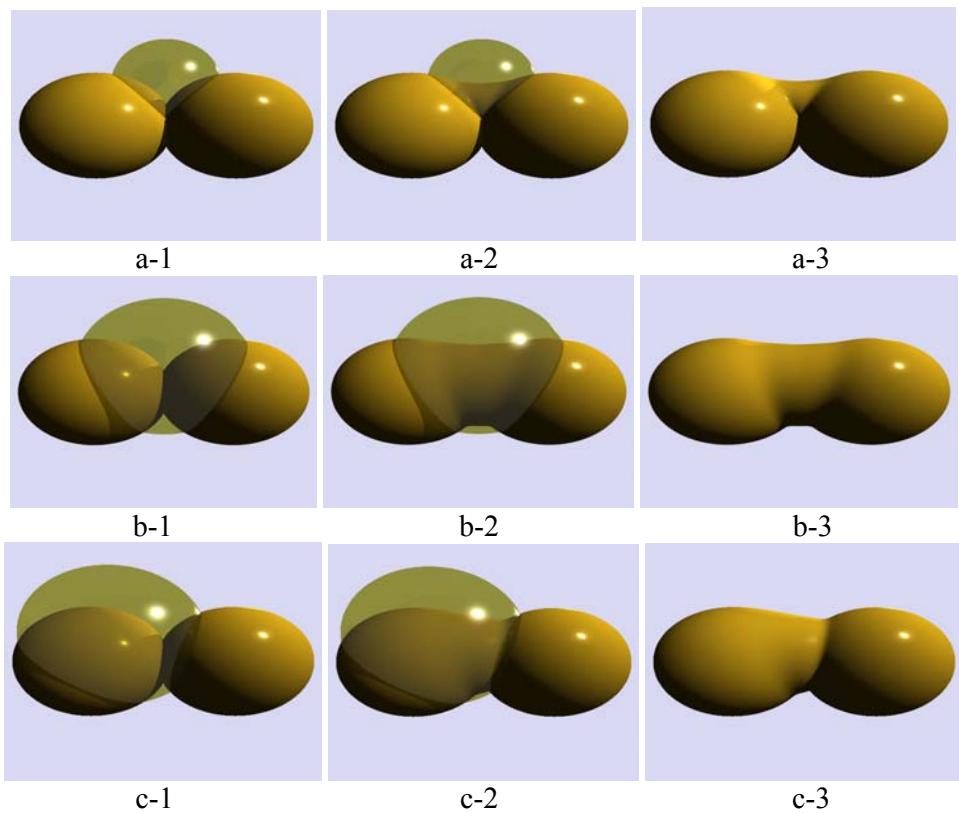


Figure 9. Shape and position of 3D blend is controlled by the bounding ellipsoid

Control of blend shape and position in 3D is illustrated by Figure 9. The pure union of two ellipsoids (Figure 9a-1) is changed to the bounded blending union using the third ellipsoid (transparent shape). The resulting blend is located strictly inside the bounding ellipsoid (Figure 9a-2), which produces an unusual blending shape localized at the top part of the initial union of ellipsoids (Figure 9a-3). At the next step, we increase the size of the bounding ellipsoid (Figure 9b) and correspondingly change the shape of the blend, which stretches out to the lower part of the initial shape, but keeps its symmetry. Then, we move the bounding ellipsoid to the left (Figure 9c) and make the blending shape asymmetric.

6.3 Multiple blending

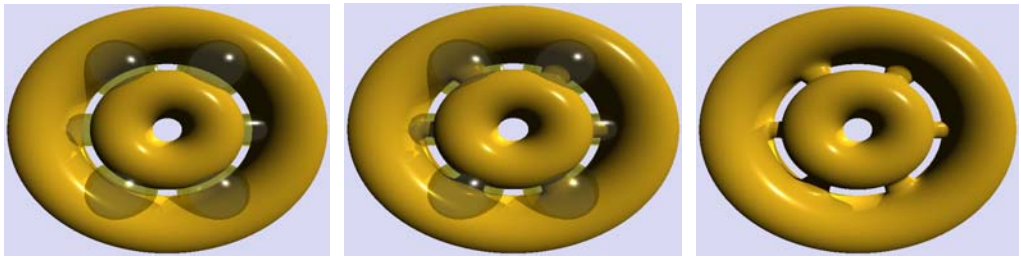


Figure 10. Multiple blending controlled by a bounding solid consisting of six disconnected components

The definition of the bounding solid by a single function allows for unusual operations such as multiple blending. As it is shown in Figure 10, the bounding solid can be constructed using arbitrary primitives and operations. In this example, the disconnected bounding solid controls the blending union of two tori. The bounding solid is described using R-functions by a single function f_3 in (Equation 8) as union of six primitives: four equally sized solid balls and two thin ellipsoids. The result of this operation is a single connected solid with multiple blending components located inside the disconnected bounding solid.

6.4 Partial edge blending

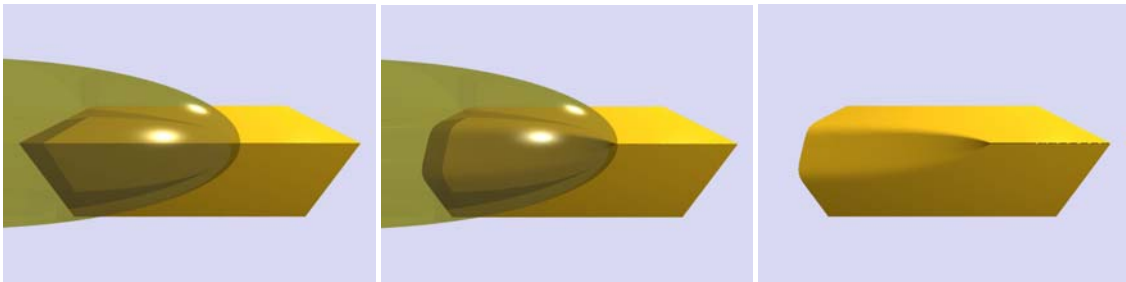


Figure 11. Partial edge blending: blending intersection of two planar halfspaces bounded by an ellipsoid overlapping with a part of the edge.

As the result of the proposed blending operation is located completely inside the bounding solid, it becomes possible to apply both blending and a pure set-theoretic operation to different parts of the initial solids. This would result in partial edge blending as it is illustrated in Figure 11. Here, blending intersection of two planar halfplanes is bounded by an elliptical solid, which overlaps only with a part of the intersection edge. As the result, we can observe the transition of the subtracted material blend to the sharp edge at the point of intersection between the edge and the surface of the bounding ellipsoid.

6.5 Blend on blend

As the proposed formulation does not restrict the initial shapes, it is possible to apply blending to the result of another blending operation. Figure 12a shows partial edge blending of Figure 11 with the

change of subtracted material blend to added material. A new elliptical shape overlapping with the blend and a new bounding solid are introduced (Figure 12b). The resulting blending is shown in Figures 12c,d. Note that the result has smooth surface transition with the initial edge as well as with the first blending surface. In principle, the order of the blending operations can be changed and a similar result can be obtained. However, to get exactly the same result and to make the blending operation completely associative, an automatic method for the evaluation of new blending parameters for a different order is needed.

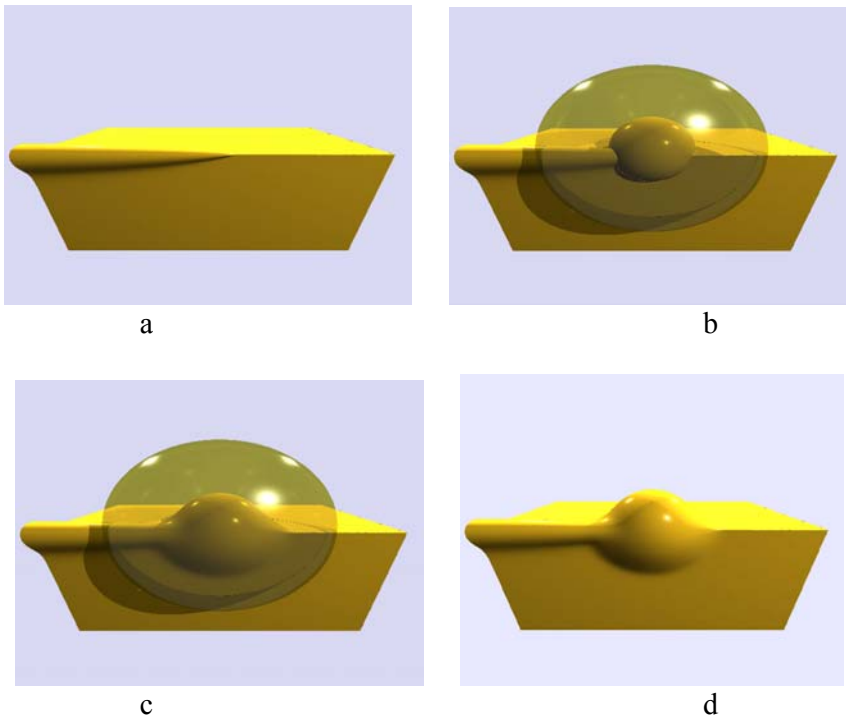


Figure 12. Blend on blend: (a) initial partial edge blend with added material; (b) new shape for blending and a transparent bounding solid; (c) blend on blend with the bounding solid; (d) blend on blend with the removed bounding solid.

6.6 Blends in constructive solids

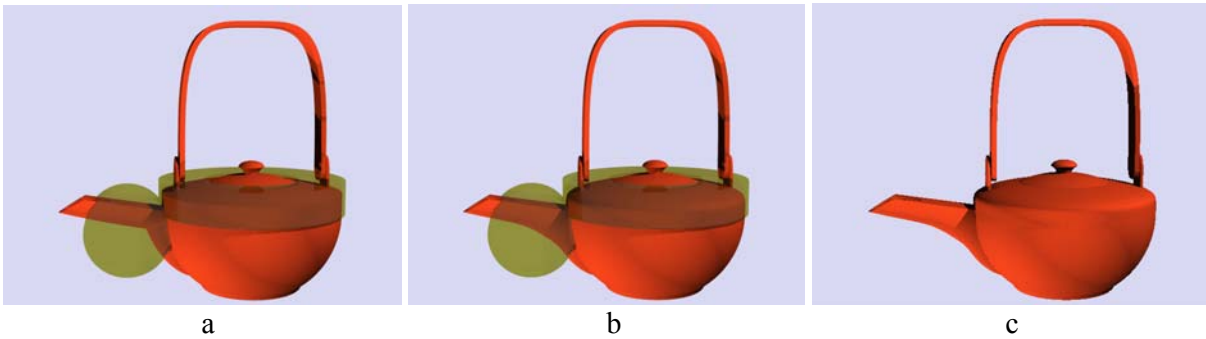


Figure 13. Two bounded blending operations are applied in the construction of a Japanese sake pot: union of the spout elements and intersection for the top part of the pot body.

The proposed bounded blending operations can replace pure set-theoretic operations in the construction of a solid without rebuilding the entire construction tree data structure. We provide in Figure 13 an example of the sake pot construction from the Virtual Shikki project [9]. Initially, the model is constructed using set-theoretic operations (see Figure 13a) with two unwanted sharp edges in the area of the spout and at the top of the pot body. For the spout, a cylindrical bounding solid is used for the blending union operation. At the top of the body, a cylinder with a hole is used for bounding the blending intersection operation. The entire object is modeled using the function representation (FRep) [8] and is defined by a single procedurally defined function of three variables. The function evaluation procedure traverses the construction tree data structure with primitives as leaves and operations as nodes of the tree. In this example, the blending operations replaced pure set-theoretic operations in the nodes of the tree with two additional subtrees added for the bounding solids. Note that the bounded blending operations have three solids as their arguments and hence require 3-ary nodes in the construction tree in comparison with binary nodes for the pure set-theoretic operations.

7. Conclusion

We introduced new analytical formulations for bounded blending set-theoretic operations. The main idea is to apply localized displacements to the standard R-functions describing pure set-theoretic operations. Two methods of blend localization were investigated. Control points on the surfaces of initial solids provide means of quite intuitive blend localization and a simple mathematical definition. However, this method does not allow a designer to select a single vertex or an edge for blending. It results in unwanted "ghost" blending along with the expected bounded blend. The second method is based on the additional bounding solid, which guarantees truly local character of the blend. The original analytical definition of this type of blending allows for using a wide variety of functionally defined solids as its arguments. This results in support of such unusual operations as multiple blending and partial edge blending, which hardly can be supported by other blending techniques.

The proposed bounded blending operations can replace pure set-theoretic operations in the solid model without rebuilding the entire construction tree data structure. It is worth noting that the operations using bounding solids have three arguments. This brings a requirement for a functionally based modeling system to support n-ary nodes in the construction tree.

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