Bounded Blending Operations

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Abstract

New analytical formulations of bounded blending for functionally defined set-theoretic operations are proposed. The blending set operations are defined using R-functions and displacement functions with the localized area of influence. The shape and location of the blend is defined by control points on the surfaces of two solids or by an additional bounding solid. The proposed blending using a bounding solid can be applied to a single selected edge or a vertex. We introduce new types of blends such as a multiple blend with the disconnected bounding solid and a partial edge blend. It is shown to have versatile applications in interactive design.

1. Introduction

A blending operation in shape modeling generates smooth transition between two surfaces. Such operations are usually used in computer-aided design for modeling fillets and chamfers. Blending versions of set-theoretic operations (intersection, union, and difference) on solids approximate exact results of these operations by rounding sharp edges and vertices. In the case of blending union of two disconnected solids, one resulting solid with a smooth surface can be generated.

Detailed discussions of blending techniques can be found in [11, 13, 1]. The major requirements to blending operations are:

- tangency of the blend surface with the initial surfaces;
- automatic clipping of unwanted parts of the blending surface;
- blending definition of the basic set-theoretic operations;
- support of added and subtracted material blends;
- intuitive control of the blend shape and position.

Let us consider the works particularly oriented to blending versions of set-theoretic operations on solids with implicit surfaces [9, 5, 8]. The blending method of [9] is based on the approximations of min/max functions used for set-theoretic operations. The proposed blending has global character in the sense that the entire initial surfaces are replaced by the blending surface. This formulation provides only added material blend for union and subtracted material blend for intersection. The work [5] provides analytical definitions for blending surfaces. However, added or subtracted material blends require additional set-theoretic operations for the appropriate truncation of the unwanted parts of the blend solid. An analytical definition of blending set-theoretic operations was proposed in [8]. This definition covered added and subtracted material blends as well as symmetric and asymmetric blending. The main problem with this approach is that the blends are global in nature and cannot be localized using the blend parameters. The local blends that control how far along a surface the blend adheres were proposed in [4, 10]. Controlled blending problem has been also addressed in implicit surface modeling research [2, 3].

In this paper, we extend the approach of [8] by introducing bounded blending operations defined using R-functions and displacement functions with the localized area of influence. The shape and location of the blend is defined by control points on the surfaces of two solids or by an additional bounding solid. The proposed blending using a bounding solid can be applied to a single selected edge or vertex. We describe several experiments and introduce new types of blends such as a multiple blend with the disconnected bounding solid and a partial edge blend. Set-theoretic operations on blends and blends on blends (i.e., recursive blends) are also supported. Therefore, the proposed operations can replace pure set-theoretic operations in the construction of a solid without rebuilding the entire construction tree data structure.

2. Blending based on R-functions

In [8], an analytical definition of blending set-theoretic operations on solids with implicit surfaces was proposed. This work was motivated by observing properties of R-functions [12, 6], which provide exact analytical definitions of set-theoretic operations with C^1 and higher-level continuity of the resulting function.

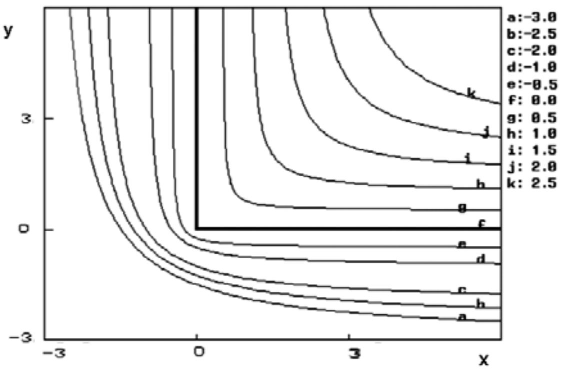


Figure 1. Contour lines of the R-function defining the intersection of halfplanes *x ≥ 0* and *y ≥ 0*.

In Fig. 1, contour lines of the function *F(x,y)* defining the intersection of halfplanes $x \geq 0$ and $y \geq 0$ are shown. Using the theory of R-functions, the result of the intersection can be described as:

$$F(x,y) = x + y - \sqrt{x^2 + y^2}$$

This function has a curve with a sharp vertex (bold line in Fig. 1) as a zero value iso-contour $f(x,y)=0$. Other contour lines are smooth in the entire domain. This property brings the idea that some displacement of the exact R-function can result in the blending effect between the lines $x = 0$ and $y = 0$.

The following definition of the blending set-theoretic operation was proposed in [8]:

$$F_b(f_1, f_2) = R(f_1, f_2) + disp_b(f_1, f_2), \qquad (1)$$

where $R(f_1, f_2)$ is an R-function corresponding to the type of the operation, the arguments of the operation $f_1(X)$ and $f_2(X)$ are defining functions of two initial solids, and $disp(f_1, f_2)$ is a Gaussian-type displacement function. The following expression for the displacement function was used:

$$disp_b(f_1, f_2) = \frac{a_0}{1 + \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2}, \qquad (2)$$

where a_0, a_1, an a_2 are parameters controlling the shape of the blend. The proposed definition is suitable for blending union, intersection, and difference, allows for generating added and subtracted material, symmetric and asymmetric blends. For example, the intersection operation can be described by an R-function as

$$R_{\text{int}}(f_1, f_2) = f_1 + f_2 - \sqrt{f_1^2 + f_2^2} \qquad (3)$$

with the corresponding blending intersection described as

$$F_{b\,\text{int}} = f_1 + f_2 - \sqrt{f_1^2 + f_2^2} + \frac{a_0}{1 + \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2}$$

The proposed displacement function does not get zero value anywhere in the space. This is the reason of the main disadvantage of this definition - the blend has global character and cannot be localized using its parameters. In this paper, we propose to use different displacement functions, which allow for localization of the blend using control points or an additional bounding solid.

3. Bounded blending

In this section, we propose new formulations for blending operations with the blend bounded by control points and by an additional bounding solid. The definition (1) of a blending operation is used as the basis with the modifications introduced only for the displacement function. First, we select a displacement function with the local area of influence (section 3.1), then, we construct a generalized distance function for bounding blends using control points (3.2) and bounding solids (3.3).

3.1. Displacement function

As the first step of defining bounded blending, we propose to select for the definition (1) a displacement function of one variable $disp_{bb}(r)$, which should satisfy the following conditions:

1) $disp_{bb}(r) \geq 0$ and $disp_{bb}(r)$ takes the maximal value for $r=0$;
2) $disp_{bb}(r) = 0, r \geq 1$

$\frac{\partial disp_{bb}}{\partial r} = 0, r = 1$ (the curve tangentially approaches axis *x* at *r=1*).

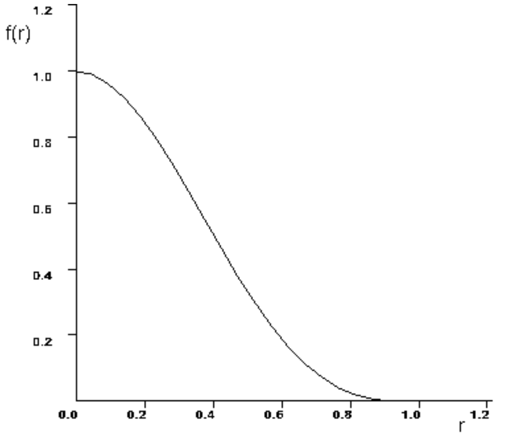


Figure 2. Displacement function for bounded blending

A plot of the desired displacement function is shown in Fig. 2. One can derive or find several analytical expressions for such function. In fact, this work has been done by many researchers for localizing individual components of skeletal implicit surfaces [1]. One of the possible expressions, which we will use in the rest of the paper, is

$$disp_{bb}(r) = \begin{cases} \dfrac{(1-r^2)^3}{1+r^2}, r < 1 \\ 0, r \geq 1 \end{cases} \qquad (4)$$

Here, *r* is a generalized distance, which is constructed using defining functions of the initial solids, as it will be shown in the next section.

3.2. Bounding by control points

We discuss here the first and possibly the simplest method of the blend localization. The idea is to place one control point on the surface of each initial solid and to require that the blend exists only "between" these control points. The control points are supposed to be relatively close to the edge resulting from the pure set-theoretic operation and it should be intuitively obvious what area is designated for blending. To satisfy such requirement, the displacement function (4) should become zero at the control points with the maximal value on or near the edge. Let us suppose that the initial defining functions $f_1(X)$ and $f_2(X)$ have distance properties. Then, the following generalized distance function can be proposed:

$$r^2(f_1, f_2) = \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2, \qquad (5)$$

where $f_1(X)$ and $f_2(X)$ are defining functions of two solids as arguments of the blending operation; $a_1 = f_1(P_2)$, where P_2 is a control point placed on the surface $f_2(X) = 0$; $a_2 = f_2(P_1)$, where P_1 is a control point placed on the surface $f_1(X) = 0$. Note the following properties of this function:

1) $r^2(0,0) = 0$;

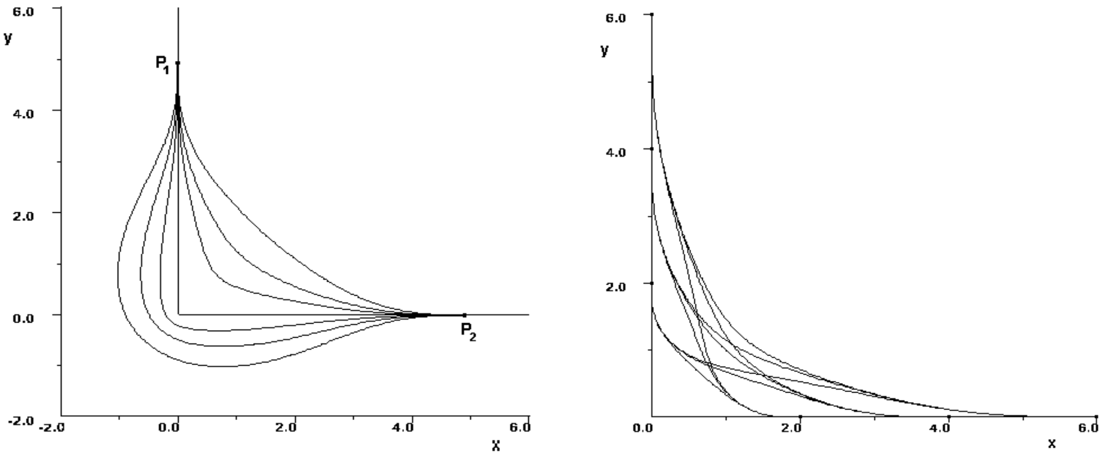
2) at the point P_1: $f_1(P_1) = 0, f_2(P_1) = a_2$, and $r^2(f_1, f_2) = 1$;

3) at the point P_2: $f_1(P_2) = a_1, f_2(P_2) = 0$, and $r^2(f_1, f_2) = 1$.

These properties mean that the displacement function $disp_{bb}(r)$ (4) takes maximal value at the intersection points of two surfaces, where $f_1(X) = 0$ and $f_2(X) = 0$, and becomes zero at control points P_1 and P_2. Using Eq. 1 and applying the displacement function $disp_{bb}(r)$ defined by Eq. 4 with r^2 defined by Eq.5, we get for the bounded blending operations with control points:

$$F_{bb}(f_1, f_2) = R(f_1, f_2) + a_0 disp_{bb}(r) \quad (6)$$

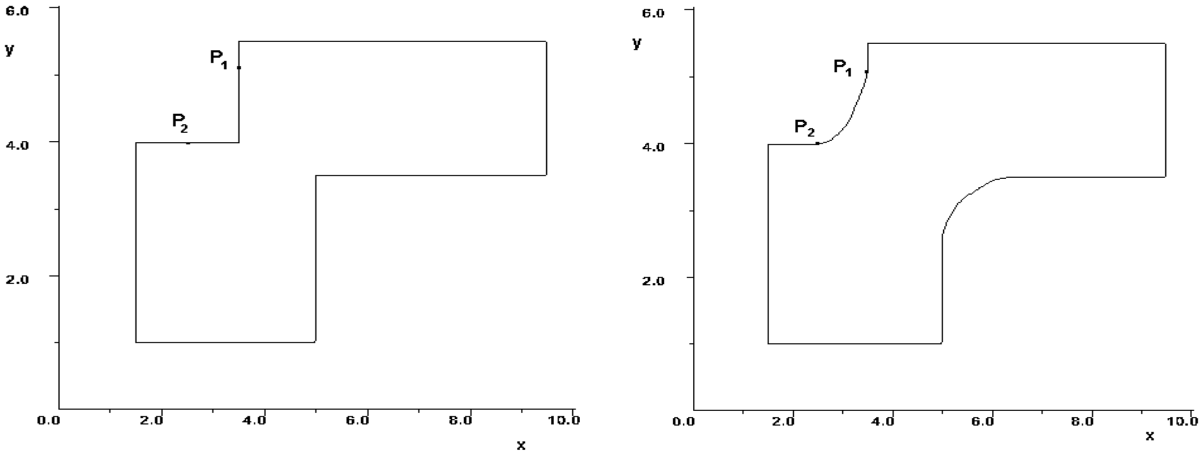
Examples of such bounded blending operations are given in Figs. 3-4. In Fig. 3, we illustrate the bounded blending intersection of two halfplanes $f_1(x, y) \geq 0$ and $f_2(x, y) \geq 0$, where $f_1(x, y) = x$ and $f_2(x, y) = y$. The control point P_1 is placed on the line $x = 0$, the boundary of the first halfplane, and the control point P_2 is placed on the line $y = 0$, the boundary of the second halfplane. One can observe that the blend is located between the control points due to the displacement function definition. In Fig. 3a, the positive values of the parameter a_0 correspond to the added material blends, $a_0 = 0$ keeps the result of the pure intersection, and the negative values of a_0 correspond to subtracted material blends. Moving control points along the initial boundaries allows for generating symmetric and asymmetric blends as it is illustrated by Fig. 3b.



a b

Figure. 3 Bounded blending intersection of halfplanes *x ≥ 0* and *y ≥ 0* with control points P_1 and P_2: added and subtracted material blends controlled by the parameter a_0; symmetric and asymmetric blends with different positions of control points.

The bounded blending union operation of two rectangles with control points is illustrated by Fig. 4. Two control points are placed on the corresponding boundaries of the rectangles (Fig. 4a). The expected bounded blend appears between the control points at the upper left part of the shape in Fig. 4b. However, the rectangles are blended also in the lower right part of the shape, which can be unexpected or unwanted. This example unveils the global character of the blend with control points, which does not allow a designer to select a single vertex or an edge for blending. Another means of defining bounds are needed to provide truly local character of blending.



a b

Figure 4. Union of two rectangles and two control points (a); expected bounded blend between control points at the upper left part of the shape (b) and unwanted blend at the lower right part of the shape (b).

3.3. Bounding solid

The introduction of an additional bounding solid can bring real local character to the blend. It is required that the blending surface exists only inside the bounding solid, and only original surfaces exist outside the bounding solid. The upper part of Fig. 5 shows two solids to be blended (defined by the functions f_1 and f_2) and a bounding solid (defined by the function f_3). To apply the bounded blending definition

$$F_{bb}(f_1, f_2, f_3) = R(f_1, f_2) + a_0 disp_{bb}(r) \quad (7)$$

with the displacement function (4), we have to provide the following interconnected properties (see lower part of Fig. 5):

1) The zero value of the displacement function $disp_{bb}(r)$ outside the bounding solid;
2) The maximal value of the displacement function $disp_{bb}(r)$ near the intersection points of two initial surfaces;
3) The zero value of the generalized distance function r at the intersection points of two initial surfaces;
4) The value 1 of the function r outside the bounding solid.

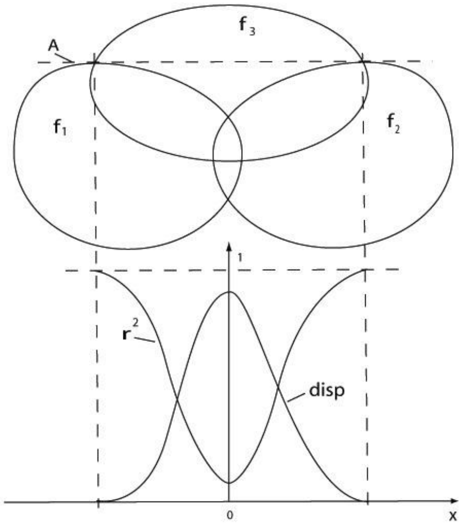


Figure 5. Components of the definition of the bounded blending union. Upper part: two solids to be blended (f_1 and f_2) and a bounding solid f_3. Lower part: behavior of the functions r^2 and *disp* in the cross-section A of the bounding solid.

The following definition satisfies the above requirements:

$$r^2 = \begin{cases} \dfrac{r_1^2}{r_1^2 + r_2^2}, r_2 > 0 \\ 1, r_2 = 0 \end{cases}, \quad (8)$$

where

$$r_1^2(f_1, f_2) = \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2,$$

and

$$r_2^2(f_3) = \begin{cases} \left(\dfrac{f_3}{a_3}\right)^2, f_3 > 0 \\ 0, f_3 \le 0 \end{cases},$$

with numerical parameters a_1 and a_2 controlling the blend symmetry, and a_3 allowing the user to interactively control the influence of the function f_3 on the overall shape of the blend. This definition of the function r and the definition (4) of the displacement function $disp_{bb}(r)$ are not unique and can be changed, if it is necessary in particular applications. We explore the properties and applications of the proposed bounded blending in the next section.

4. Experiments and applications

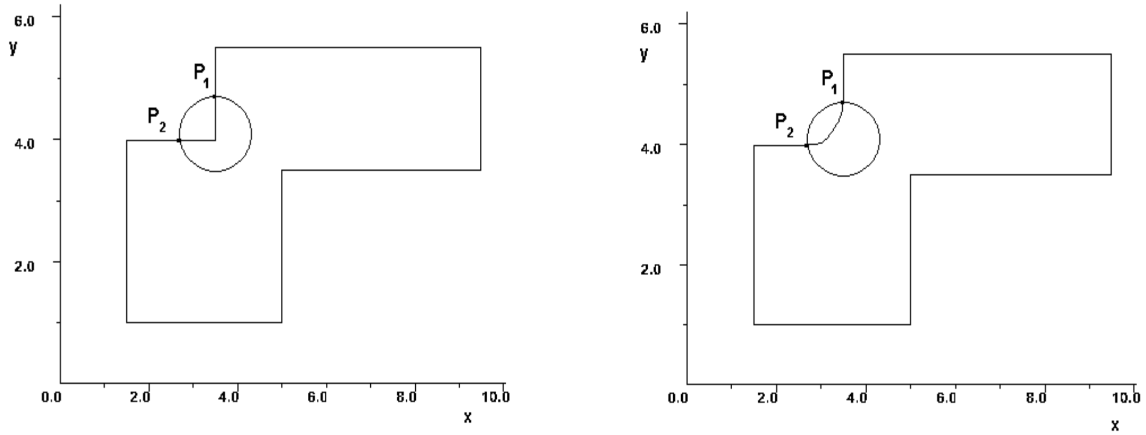
In this section, we illustrate such properties of the proposed blending operations with bounding solids as their local character and intuitive control of blend shape and position. The analytical definition of blends allows for support of such unusual operations as multiple blending and partial edge blending.

4.1. Feature selection for blending

The use of a bounding solid allows the user to select a single feature (vertex or edge) of the constructive solid for blending. The corresponding pure set-theoretic operation should be replaced by the bounded blending operation. In Fig. 6, we present an example of union of two rectangular solids in 2D. A 2D disk is used as a solid bounding the blend in the area near a single vertex (Fig. 6a). The pure union operation is replaced by the bounded blending union (7) and the result is shown in Fig. 6b. The expected bounded blend appears inside the bounding disk in the upper left part of the shape in Fig. 6b. However, in comparison with Fig. 4b, no blending appears in the lower right part of the shape. This example illustrates the local character of the blending with a bounding solid.

4.2. Control of blend shape and position

The shape and position of the blend is controlled by its parameters and by the position and shape of the bounding solid. A family of blends inside the bounding ellipse is shown in Fig. 7. Subtracted material blends are described by Eq. 7 with negative a_0 values, added material blends correspond to positive a_0.



a

b

Figure 6. Blending union at a single vertex selected by the bounding disk.

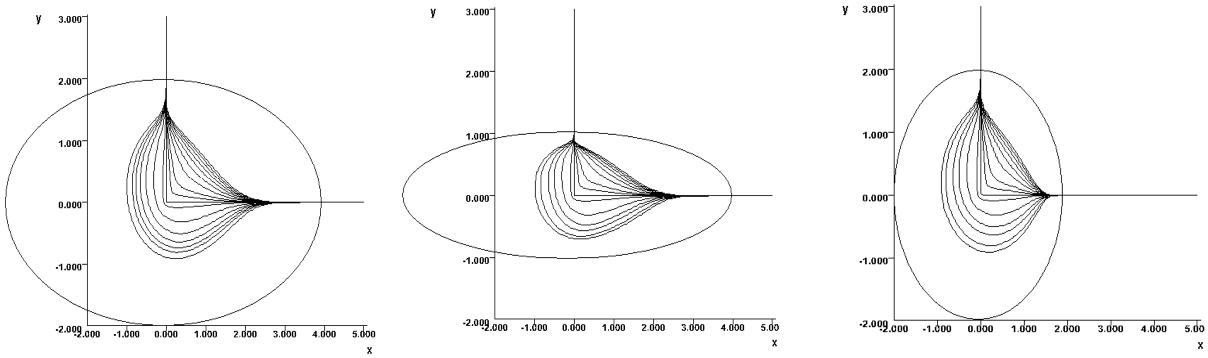
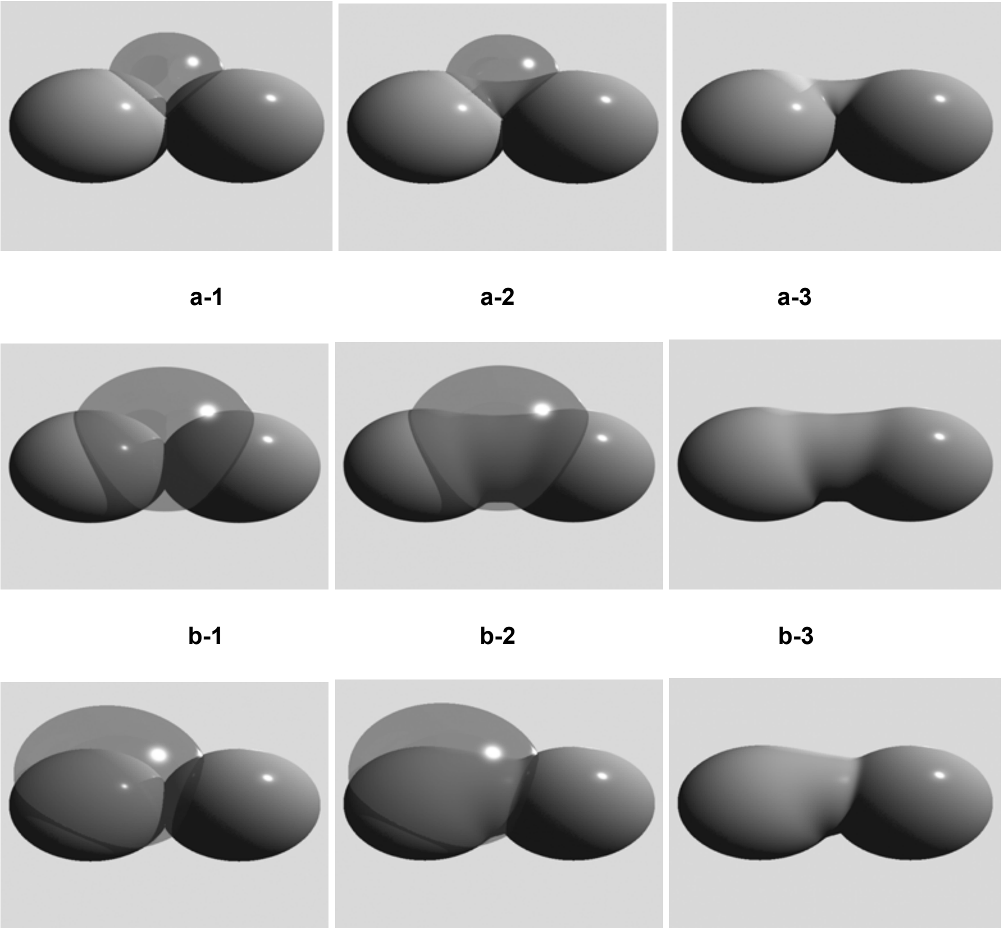


Figure 7. Shapes of 2D blends are controlled by changing parameters of the bounding ellipsoid.

Fig. 7 also shows that the blend changes its shape following changes of the bounding solid. Control of blend shape and position in 3D is illustrated by Fig. 8. The pure union of two ellipsoids (Fig 8a-1) is changed to the bounded blending union using the third ellipsoid (transparent shape). The resulting blend is located strictly inside the bounding ellipsoid (Fig. 8a-2), which produces an unusual blending shape localized at the top part of the initial union of ellipsoids (Fig. 8a-3). At the next step, we increase the size of the bounding ellipsoid (Fig. 8b) and correspondingly change the shape of the blend, which stretches out to the lower part of the initial shape, but keeps its symmetry. Then, we move the bounding ellipsoid to the left (Fig. 8c) and make the blending shape asymmetric.

4.3. Multiple blending

The definition of the bounding solid by a single function allows for unusual operations such as multiple blending. As it is shown in Fig.9, the bounding solid can be constructed using arbitrary primitives and operations. In this example, the disconnected bounding solid controls the blending union of two tori. The bounding solid is described using R-functions by a single function f_3 in (8) as union of six primitives: four equally sized spheres and two thin ellipsoids. The result of this operation is a single connected solid with multiple blending components located inside the disconnected bounding solid.



a-1 a-2 a-3

b-1 b-2 b-3

c-1 c-2 c-3

Figure 8. Shape and position of 3D blend is controlled by the bounding ellipsoid

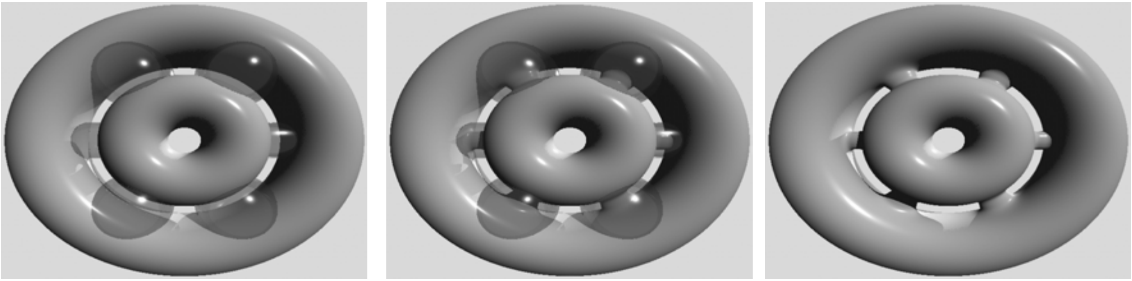


Figure 9. Multiple blending controlled by a bounding solid consisting of six disconnected components

4.4. Partial edge blending

As the result of the proposed blending operation is located completely inside the bounding solid, it becomes possible to apply both blending and a pure set-theoretic operation to different parts of the initial solids. This would result in partial edge blending as it is illustrated by Fig. 10. Here, blending intersection of two planar halfplanes is bounded by an elliptical solid, which overlaps only with a part of the intersection edge. As the result, we can observe the transition of the subtracted material blend to the sharp edge at the point of intersection between the edge and the surface of the bounding ellipsoid.

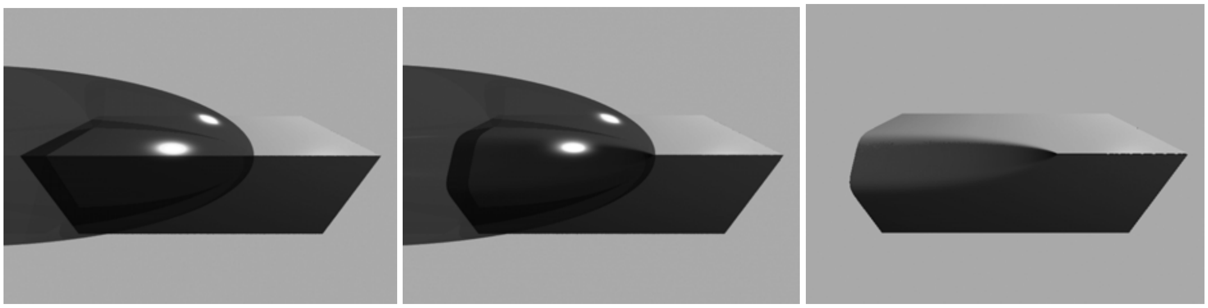


Figure 10. Partial edge blending: blending intersection of two planar halfspaces bounded by an ellipsoid overlapping with a part of the edge

4.5. Blends in constructive solids

The proposed bounded blending operations can replace pure set-theoretic operations in the construction of a solid without rebuilding the entire construction tree data structure. We provide in Fig. 11 an example of the sake pot construction from the Virtual Shikki project [7]. Initially, the model is constructed using set-theoretic operations (see Fig. 11a) with two unwanted sharp edges in the area of the spout and at the top of the pot body. For the spout, a cylindrical bounding solid is used for the blending union operation. At the top of the body, a cylinder with a hole is used for bounding the blending intersection operation. The entire object is modeled using the function representation (FRep) [6] and is defined by a single procedurally defined function of three variables. The function evaluation procedure traverses the construction tree data structure with primitives as leaves and operations as nodes of the tree. In this example, the blending operations replaced pure set-theoretic operations in the nodes of the tree with two additional subtrees added for the bounding solids. Note that the bounded blending operations have three solids as their arguments and hence require 3-ary nodes in the construction tree in comparison with binary nodes for the pure set-theoretic operations.

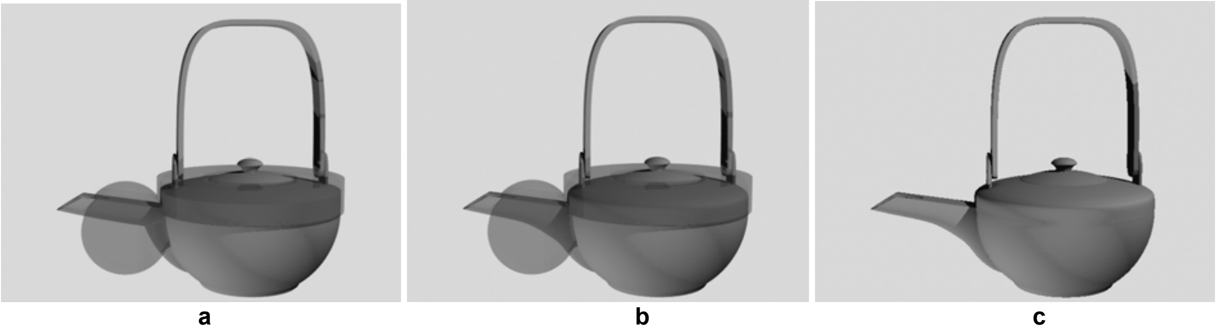


Figure 11. Two bounded blending operations are applied in the construction of a Japanese sake pot: union of the spout elements and intersection for the top part of the pot body.

5. Conclusion

We introduced new analytical formulations for bounded blending set-theoretic operations. The main idea is to apply localized displacements to the standard R-functions describing pure set-theoretic operations. Two methods of blend localization were investigated. Control points on the surfaces of initial solids provide means of quite intuitive blend localization and a simple mathematical definition. However, this method does not allow a designer to select a single vertex or an edge for blending. It results in unwanted "ghost" blending along with the expected bounded blend. The second method is based on the additional bounding solid, which guarantees truly local character of the blend. The analytical definition of this type of blending allows for using a wide variety of functionally defined solids as its arguments. This results in support of such unusual operations as multiple blending and partial edge blending.

The proposed bounded blending operations can replace pure set-theoretic operations in the solid model without rebuilding the entire construction tree data structure. It is worth noting that the operations using bounding solids have three arguments. This brings a requirement for a functionally based modeling system to support n-ary nodes in the construction tree.

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