

Functionally-based Augmented Sculpting

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Abstract

In this paper we describe an approach to computer-aided sculpting concerned with the creation and modification of digital models based on physical abstract sculptures. We begin by presenting a survey of current methods for the creation of computer-aided sculptured artefacts. Then we proceed to present some original methods and tools based on the Function Representation of geometric models. We introduce a specialised computer language, named HyperFun, which facilitates the modelling of complex objects. As well as presenting computer-generated animated models of pre-existing sculptures by Russian artist Igor Seleznev, we also show how some interesting novel shapes can be generated using the HyperFun system. Finally we outline two advanced projects concerned with creating a sculpture-based augmented reality which allows for the interactive participation of the observer. In this paper, we present experimental results, which hopefully have some artistic appeal. These results were produced by an international team of researchers and students collaborating through the Internet.

KEYWORDS: Virtual sculpting, augmented reality, function representation, volumetric models, animation, metamorphosis.

Introduction

Since the inception of computing the tantalising possibility that one day computers will be able to produce artefacts automatically without human intervention has been advanced by a number of artists, scientists and Science Fiction writers. The human desire to communicate with other sentient beings, even beings of our own creation,

that exhibit an alien type of intelligence and creativity has fuelled research in artificial intelligence and remains a fascinating research question. More realistically, however, the use of computer hardware and software to aid the artist in the creative process can be seen as a set of sophisticated tools, analogous to the use of the brush and canvas or the chisel and marble, leading to machine mediated augmentation of human creativity.

In this paper we focus our attention on the area of computer-aided sculpture and carving, which is a human activity aimed at producing new shapes by cutting or engraving some initial shape (lump of material). Currently, the computer can aid the full cycle from sketching the initial idea of a sculpture and exploring a variety of different shapes all the way to the physical production of the final sculpture. Existing methods and tools as well as known works in computer-aided sculpture and carving are discussed in the following section.

In this paper, we present a new approach. Starting with a physical sculpture, we produce its computer model and we interactively manipulate this model to generate new shapes that can eventually be manufactured to produce a new physical sculpture. We call this approach "augmented sculpting", as it extends the existence of physical artefacts to a virtual computer-world and then closes the loop bringing new computer models into physical existence.

The technology on which this work is based is known as the Function Representation (FRep) of geometric models. The original mathematics, algorithms and tools that we use have been developed and continue to be developed by the HyperFun project team. HyperFun is an international free software project on Functionally Based Shape Modelling, Visualisation and Animation (<http://www.hyperfun.org>). Members of the

HyperFun Team – a freely associated group of researchers and students from different countries – have contributed to the Augmented Sculpture study presented in this paper.

Related Works

In this section we present a concise review of the technological and artistic aspects of related works concerned with computer-aided sculpting.

Technological Aspects

There are two main concerns in the creation of the computer model of a sculpture, namely the way we model its shape and the way we represent this model. Shape design can be performed in various ways: using 2D/3D input with subsequent reconstruction, using a purely mathematical description, using an interactive modelling technique that employs pressure-sensitive or haptic interactions, or alternatively using, a set of virtual reality interface tools (such as cybergloves and a head-mounted display). The presentation of the modelling results can be attained by hand-manufacturing the final sculpture from physical materials, by utilising rapid prototyping machinery to produce the physical objects, or in the form of still images, animations, or virtual reality presentations.

From the technological point of view, tools for computer-aided sculpting were described by Keskeys¹ as "a system that will allow the reshaping of illusory computer material by carving into it, adding material to it or modelling it would open new and exciting creative opportunities". Since then, computer-aided sculpting has become one of the central themes in computer graphics literature and a number of techniques have been developed that facilitate the construction and modification of sculptural shapes².

Such techniques employ a variety of sculptural metaphors. When it comes to using 2D input devices for the approximation of 3D shapes, the original techniques for rapid design are implemented using many diverse approaches. For instance, the Sketch system³ facilitates the construction of 3D shapes using CSG techniques and in the Teddy system⁴, which is pen-based, the user is allowed to enter free-form strokes that the system interprets to deduce the form of the 3D shape and to generate a polygonal model.

To deal with 3D sculptural shapes, global deformations^{5,6} can be used. These deformations have few control parameters and are easy to handle by the user. Because of their non-locality, however, they are not very suitable for surface sculpting. Alternatively, local deformations⁷⁻¹⁰ based on Free-Form Deformations with multiple control points can provide very complex continuous shape control. These deformations, however, are computationally expensive and it is frequently cumbersome for the user to specify and modify the control points of the deformation lattice.

Another approach is based on interactive local modifications of a basic shape which are applied sequentially one after another. This is often referred to as the “sculpting metaphor”. This metaphor utilises a modelled virtual carving tool (chisel) and such operations as set-theoretic operations, offsetting, and blending are usually available. This approach has been used for voxel modelling¹¹⁻¹⁶, Constructive Solid Geometry (CSG) modelling¹⁷, parametric modelling¹⁸⁻²⁰ and functionally-based modelling^{21,22}. Also worth mentioning is an interesting deformation technique called “wires”²³ that is

based on implicit models and was inspired by the “armatures” sculpting metaphor. This deformation technique has been implemented in the Maya animation system.

The metamorphosis between a number of given geometric shapes can be a useful technique for generating interesting new shapes. Given two volumetric objects, scheduled Fourier metamorphosis²⁴ and feature-based metamorphosis²⁵ have been proposed. The production of evolutionary 3D aesthetically pleasing shapes²⁶ is also a promising direction for computer-aided sculpting which requires further research and experimentation. Finally, one should mention the Digital Clay Project which aims to provide a finger sculpting metaphor based on a novel distributed input / display device whose surface can be shaped by the user and captured by the computer and vice versa²⁷.

Computer-aided Sculpting as Art

In artistic terms, there is the variety of styles in computer-aided sculpture represented in the works of known computer researchers and artists. Let us mention a few of these.

Carlo Sequin²⁸ is a computer scientist experimenting with modelling unusual shapes using parametric and subdivision surfaces. He produces these shapes in wood or bronze by hand or using stereolithography rapid prototyping. He works closely with Brent Collins, who is a professional wood sculptor producing physical versions of the computer-designed shapes by hand-cutting shape slices from wood board, laminating and finely polishing the sculpture.

Helaman Ferguson²⁹ sees himself as both a mathematician and a sculptor. Borrowing ideas from mathematics he produces stone sculptures. His computer tool position and

orientation monitoring system does not do the cutting work, it just provides quantitative information during the hand cutting process.

Bruce Beasley³⁰ is a sculptor who has been using computer-aided design tools to experiment with three-dimensional polygonal shapes and to transfer the elements of the finally selected model to foam core (a piece of polystyrene sandwiched between two sheets of paper) using a plotter. The foam core components are used as patterns for bronze plates that are joined together to form a sculpture.

Stewart Dickson³¹ is both an artist and a computer specialist. He experiments with different mathematical and computer graphics software and produces the computed shapes using stereolithography and other rapid prototyping equipment. In his recent works, he used a 3D colour-printer to produce volumetric objects with internal colour distribution.

Interactive Sculpting Projects

A number of sculpture-related integrated projects are described in the literature. They demonstrate how one can combine different artistic methods and technical tools to generate sculptural artefacts. The projects we describe here are very difficult to compare as they exploit very different methods, techniques, tools and user-interaction metaphors. However they give us a flavour of the breadth and diversity of the work undertaken in virtual sculpting.

An interactive volumetric sculpting project was implemented by Eric Ferley et al¹⁶. The sculptor deals with a volumetric model based on isosurfaces over a sampled scalar field, and user-defined free-form tools allow for the adding/removal of material with subsequent smooth local deformations. The tools are used as a stamp to

make imprints on an existing shape thus using the virtual clay metaphor. Visual feedback is implemented through the use of a high-quality stereoscopic display. Haptic feedback with an articulated arm, such as the PHANTOM, is also possible.

William Latham is an artist who in collaboration with computer scientist Stephen Todd has implemented a suite of interactive tools encompassing the entire computer-based process for the creation of evolutionary art forms. In their approach the sculpting process starts from the initial concept and hand-drawn sketches, proceeds through to the definition and application of their distinctive artistic “evolutionary” style and culminates with the rendering of three-dimensional forms and the creation of gallery artworks and animations²⁴.

Alexei Sourin²² has implemented a virtual embossing system for virtual copper plates embossing and realistic rendering of the computed results. He uses a pressure-sensitive tablet for deeper immersion of the user and closer imitation of the real embossing process. His system employs direct editing operations on a functionally-based object. It is worth noting that Sourin uses a functional representation of geometric shapes similar to the one used in our work, which is described in the following section.

The distinctive feature of an interactive sculpture environment developed by Kevin McDonnell³² is the integration of the geometry of sculptured free-form solids with the physically-based modelling framework which augments geometric entities with material properties and physical behaviours. The user can interact with the subdivision-based virtual clay through a 3D input device. In this paper we also

describe a sculpting environment which models material properties with dynamic features.

The Genetic Sculpture Park project³³ has been implemented as an interactive JAVA/VRML world and is available through Web-based interaction. The project seeks to engage the artist and observers in a creative dialogue and to empower the novice in the creation of complex sculpting models inside a virtual 3D environment. Sculptures are described as recursive tree structures with nodes containing genetic information. The user can customise the sculptures in the park based on previous likes and dislikes by employing genetic algorithms. In this paper we present an Augmented Sculpture Installation project which allows observers to be immersed into a multidimensional virtual sculpture world.

Functionally-based Methods and Tools

In this section, we provide a brief introduction into the functionally based methods and tools which provide the technical underpinning for the implementation of our Augmented Sculpture study.

Function Representation

The idea of representing art works by analytical mathematical functions can appear rather strange and impractical to people from the art world. Our approach is to provide shape-modelling tools for computer art and is based in a unified manner on the Function Representation (FRep) of the underlying model³⁴. This representation is a generalisation of traditional implicit surfaces and constructive solid geometry (CSG).

Mathematically speaking, an FRep defines a geometric object by an inequality of the form $F(X) \geq 0$, where $X = (x_1, x_2, \dots, x_n)$ is a vector representing a point in n -dimensional Euclidean space.

This modelling approach makes it possible to represent with a single continuous function such diverse entities as traditional skeleton-based implicit surfaces, convolution surfaces, constructive solids (using the so-called R-functions for set-theoretic operations), swept objects, and volumetric objects. The advantage of this modelling representation is that the result of any supported operation can be treated as input for a subsequent operation, thus very complex models can be created in this way from a single functional expression. When using FRep there is no distinction made between soft objects, CSG solids, and volumetric (voxel) objects, which are all processed in the same manner. This allows us to solve such long-standing problems as metamorphosis between objects of differing topologies, sweeps generated by a moving solid, controlled blending for all types of set-theoretic operations, collision detection and hypertexturing for arbitrary solids, direct modelling of space-time, multidimensional objects, etc.

A more general “Hypervolume” model has been introduced³⁵ which allows for the modelling of multidimensional point sets with attributes. A point set is an FRep-based geometric model of a real object. An attribute is a mathematical model of an object’s property with an arbitrary nature (such as, material, photometric, physical etc.). This modelling approach is general enough to cover objects in solid modelling, heterogeneous objects modelling and volumetric modelling.

The HyperFun Language and Tools

HyperFun³⁶ is a specialised high-level language which allows for a parameterised description of functionally-based multidimensional geometric shapes. While being minimalist in design and easy to master (our extensive experience shows that half an hour is enough for an undergraduate student to start working without further guidance), it supports all main concepts of FRep. This language was designed to be as simple as possible in order to allow non-specialist users to create models of complex geometric shapes.

A model in HyperFun can be constructed using assignment statements, conditional selection statements and iterative statements. Functional expressions are composed using conventional arithmetic and relational operators. It is possible to use standard mathematical functions as well as special built-in operators, which support fundamental set-theoretic operations.

In principle, the language is self-contained and allows users to build objects from scratch, without the use of any pre-defined primitives. However, its expressive power is greatly increased by the availability of the system “FRep library” that is easily extendable, can be adapted to particular application domains and can even be customised for the needs of particular users.

Currently, the version of the FRep library that is in general use contains the most common primitives and transformations of quite a broad spectrum. Thus, there are functions implementing conventional CSG primitives (e.g. block, sphere, cylinder, cone, and torus), popular implicits (i.e. blobby objects, soft objects, metaballs) including the recently implemented convolution objects with skeletons of different types (i.e. points, line segments, arcs, triangles, curves, and meshes). Primitives

derived from parametric functions (i.e. cubic spline and Bézier objects) have also been implemented recently. The usual primitive transformation operations are available, such as rotation, scaling, translation, twisting, stretching, tapering, blending union/intersection, as well as some more general operations such as non-linear space mappings driven by arbitrary control points.

Let us briefly describe the following three software tools that are freely available and can be downloaded from the HyperFun Project Web site (www.hyperfun.org).

- The “HyperFun Polygonizer” is a program that polygonizes and displays an object input from a HyperFun file. Support for higher dimensional models is also available. The program also makes it possible to output the results in VRML format.
- “HyperFun for POVray” is a plug-in to the popular ray-tracer POVray (www.povray.org) which makes it possible to generate high quality photorealistic images on an ordinary PC. Animation capabilities are also available.
- “HyperFun for Windows” is an interactive system allowing the user to easily master the FRep modelling concepts using the HyperFun language while working in a conventional MS Windows environment. This program allows the user to specify an FRep model in the HyperFun language, to compose complex scenes with multiple objects, to specify visual parameters for subsequent rendering and to generate animations.

The Augmented Sculpture Project

In this section, we present a detailed description of a sculpture-based case study, which allows us to demonstrate the use of a variety of functionally-based modelling techniques available to an artist using the HyperFun toolkit.

Modern abstract sculpture is very much concerned with the exploration of novel non-trivial shapes and we believe that the computer technology being presented here can assist artists in their creative quest. Functional representation methods actually employ constructivist techniques that allow the emulation of physical or virtual “building blocks” in the form of geometric primitive shapes that can be combined in complex spatial relationships. A computer-based means of sculptural representation along with a specific environment where the sculptural shapes are set can lead to the production of artefacts with a new aesthetic. Consequently, viewers experiencing these shapes within a virtual space can also benefit from this technology.

Our case study is based on real (physical) sculptures created by the well-known Russian artist Igor Seleznev (images of his work can be found at www.russian-art.org). In 2000 he became the artist in residence for the HyperFun project. Since then he has collaborated with the HyperFun team on a regular basis.

From Physical to Virtual Sculptures

In our discussion we will examine the three “real” sculptures illustrated in Figure 1.

These are:

- “Naked”, which is a 28 cm high bronze statuette (shown in Figure 1a);

- “Plasticity 3” or “Gymnast”, which is a 22 cm in high a bronze statuette (shown in Figure 1b);
- “Walking Androgynous”, which is a 110 cm in high marble statue (shown in Figure 1c).

First, it was necessary to construct the geometric models of those physical sculptures. All three selected sculptures have quite complex shapes with subtle features. Modern mathematics provides a variety of functions for modelling very complex non-regular forms. In particular, the so-called convolution surfaces are very attractive in this respect. Broadly speaking, convolution surfaces^{37,38} are a class of implicit surfaces allowing us to model complex free-form shapes. They are defined as the integral of a potential (field) function over a skeleton in the form of points, lines, arcs, and triangles. Field functions with their superposition property make it possible to model very smooth and non-regular shapes. One would imagine that modelling shapes with such functions would be quite complex. The FRep library, however, contains a number of predefined functions that only require the user to specify a number of intuitively clear parameters, thus greatly facilitating the modelling process without requiring the user to have a good understanding of the underlying mathematics.

The geometric models of the sculptures have been described in the HyperFun language. Figure 2 shows a set of photorealistic renderings of the geometric models of the aforementioned sculptures. The FRep models for these sculptures were created by students of the Moscow Engineering Physics Institute.

- “Naked” was modelled by A. Dyakonov and E. Krivochenko using the following FRep functions: ellipsoids (hfEllipsoid), convolution surfaces (hfConvPoint,

hfConvLine), affine transformations (hfShift3D, hfScale3D, hfRotate3D), blending intersection and union operations (hfBlendInt, hfBlendUni).

- “Gymnast” was modelled by P. Yablochkin and N. Varivoda using basic geometric primitives defined by equations as well as by the following set of FRep functions: non-linear deformations (hfBending3D), affine transformations (hfShift3D, hfScale3D, hfRotate3D) and blending (hfBlendInt, hfBlendUni).
- “Walking Androgynous” was modelled by S. Berzin and S. Sustavov using the following FRep functions: convolution surfaces (hfConvLine) and blending union (hfBlendUni).

Model building and debugging was carried out using the “HyperFun for Windows” toolkit, and the final ray-traced renderings (shown in Figure 2) were generated using the “HyperFun for POVray” toolkit.

The Animation of Sculptures

Once the model of a physical sculpture is created, one of the most natural things that the artist may wish to do would be to alter the material that would have been used to create the corresponding physical sculpture and to explore the visual appeal of such alternatives. To achieve this, within the functional framework, we may employ a texturing method based on the Hypervolume modelling techniques. In a previous paper³⁹ we have applied such a technique to the model of the "Naked" sculpture.

What would also be quite interesting for an artist to do and, obviously, impossible without the mediation of computer technology, is to have the ability to animate a

sculpture which being initially still, has internal (although hidden) dynamics and therefore a natural propensity to move.

In computer animation, traditionally, there is an artificial destination made between the modelling and the animation processes that leads to limitations of the forms that can be animated. The advantage of FRep is that one can work with both the shape and its animation simultaneously. So, time-dependent shapes can be modelled directly as four-dimensional (4D) entities with the subsequent generation of time cross-sections (3D objects) taken at discrete values along the time dimension. Thus, a 4D “space-time” FRep model defined by the inequality $F(x,y,z,t) \geq 0$ provides a natural solution to this problem.

Among the three sculptures that we have examined so far, “Gymnast” appears to be the most suitable candidate for animation. Alexander Ogarko redeveloped the geometric model of this sculpture and introduced ten parameters that were intended to aid the animation process. Figure 3 shows a few frames from an animation sequence which was generated using the “HyperFun for POV-Ray” toolkit. The entire sequence can be downloaded from the following Web address: <http://cis.k.hosei.ac.jp/~F-rep/App/ASP/FASP.html>.

Multidimensional Modelling for Shape Metamorphosis

4D “space-time” modelling is just a particular case of multidimensional functionally-based modelling that allows us to accomplish non-trivial effects while modelling and animating. One important application of this type of modelling, which is of great interest to both artistic and commercial animators, is the problem of shape metamorphosis. Traditionally this is a complex problem that requires the animator to

establish a set of correspondences between the initial and final key-shapes of a metamorphosing object. This process is quite cumbersome and it is incapable of dealing with key-shapes of differing topologies. In addition, using the traditional approach it is difficult to obtain intermediate shapes by interpolating between more than two key-shapes. The HyperFun toolkit supports an original technique that allows us to successfully solve the metamorphosis problem using FRep to represent the key-shapes.

In the FRep framework, metamorphosis is performed almost trivially. This framework is capable of handling key-shapes of differing topology and of generating intermediate shapes that exhibit genus change and may be composed of disjoint components. In this framework, a multidimensional object defined in an abstract n -dimensional geometric space is given by an inequality of the form $F(x_1, x_2, \dots, x_n) \geq 0$. In parallel with this “abstract world” there exists another “real world” which is populated by “multimedia objects”. A multimedia object can be seen as a multidimensional object with a set of 2D or 3D geometric coordinates (e.g. Cartesian, cylindrical, etc.), a set of dynamic coordinates (e.g. physical time), a set of photometric coordinates (e.g. colour, transparency, texture, etc.) and a set of audio or other “multimedia coordinates”. Both “abstract coordinates” and “multimedia coordinates” may assume values within given intervals.

The essence of our approach⁴⁰ lies in introducing a space mapping between geometric coordinates and multimedia coordinates that effectively establishes a correspondence between an abstract multidimensional shape object and a real multimedia object. The HyperFun toolkit supports such a mapping with a variety of multimedia types. The

“HyperFun for Windows” interactive system provides the user with a convenient way of defining such a mapping through an interactive interface.

To perform a non-trivial metamorphosis where more than two objects are involved, we have introduced special types of multimedia coordinates called the “spreadsheet coordinates”. This spreadsheet-like coordinate system requires us to organise a set of 1D, 2D, or 3D nodes, consisting of elementary images or shapes, in some sort of tabular arrangement. For example, a rectangular arrangement of these nodes gives rise to a “rectangular spreadsheet” metamorphosis scheme, which in mathematical terms is defined as a bilinear interpolation of the four key-shapes situated at the corners of the rectangle. Different topological arrangements of the nodes can be found in Fausett et al⁴¹.

A similar approach was used to handle the three sculptures discussed above. Having generated the 3D geometric models of the three sculptures “Naked”, “Gymnast” and “Androgynous”, the dimensionality of an integral “metamorphosis model” was increased to 5D to include two additional coordinates x_4 and x_5 , which were mapped onto the polar coordinates (r, α) . As we wished to produce new shapes by interpolating the three initial key-shapes of the sculptures, we introduced a “triangular spreadsheet” interpolation scheme that utilised polar coordinates. In this arrangement, the corners of the triangular spreadsheet are the initial key-shapes of the sculptures. Thus, an intermediate shape of the metamorphosis process, at any given point inside the triangle, is represented by a weighted sum of the defining functions of the initial shapes. The position of the given point is determined by its polar coordinates (corresponding to the x_4 and x_5 coordinates in our model) with the origin at the base triangle centre. The

individual weight of each sculpture in the intermediate shape is composed of two factors. To define the first factor, we introduce a “sculpture triangle” (see Figure 4 where the i^{th} sculpture corresponds to the i^{th} “sculpture triangle”). Then, the first weighting factor is specified as the ratio of the areas of “the sculpture triangle” and the base triangle. The second factor is introduced to normalise the defining functions, so as to balance their influence on the intermediate shape. The complete model in the HyperFun language is given by the following code:

```
Meta5D_triangle(x[5],a[3]){
-- balancing factors a[1], a[2], a[3] are defined experimentally
-- d1, d2, d3 - distances from sculptures to the point of interest
d_An =sqrt(1+x[4]^2 -2*x[4]*cos(3.14/3 - x[5]));
d_Na =sqrt(1+x[4]^2 -2*x[4]*cos(3.14 - x[5]));
d_Gy =sqrt(1+x[4]^2 -2*x[4]*cos(3.14/3 + x[5]));
-- Base equilateral triangle: side and area
Side_Triangle=sqrt(3);
Area_Triangle=3*sqrt(3)/4;
-- “sculpture triangles”: half-perimeters and areas
p_An=(d_Na+d_Gy+Side_Triangle)/2;
area_An=sqrt(p_An*abs(p_An-d_Na)* abs(p_An-d_Gy)*abs(p_An-
Side_Triangle));
p_Na=(d_An+d_Gy+Side_Triangle)/2;
area_Na=sqrt(p_Na*abs(p_Na-d_An)* abs(p_Na-d_Gy)*abs(p_Na-
Side_Triangle));
```

```

p_Gy=(d_An+d_Na+Side_Triangle)/2;

area_Gy=sqrt(p_Gy*abs(p_Gy-d_An)*abs(p_Gy-d_Na)*abs(p_Gy-
Side_Triangle));

-- wighting factors

w_An=(area_An/Area_Triangle)*a[1];

w_Na=(area_Na/Area_Triangle)*a[2];

w_Gy=(area_Gy/Area_Triangle)*a[3];

-- final shape Meta5D_triangle(x1,x2,x3,x4,x5)

Meta5D_triangle = w_A*Androgynous(x,a) + w_N*Naked(x,a) +
w_G*Gymnast(x,a);

}

```

Here, Naked(), Gymnast(), Androgynous() are calls to functions representing the sculpture models, which are assumed to have been defined before their use in the Meta5D_triangle() function body.

Figure 5 shows a number of intermediate shapes at different points inside the triangle, produced by metamorphosing between the three initial key-shapes. Each image was produced using direct ray-tracing of the shape defined by the HyperFun model using the “HyperFun for POVray” toolkit. The most important aspect of this approach is that the artist can use it to discover novel shapes starting from a series of key-shapes. The artist may only have a fairly vague idea of the shapes sought after and may discover quite unexpected interesting shapes by progressive refinement. A series of metamorphosed shapes generated in this manner were assessed by our resident artist

who was quite impressed by their unusual forms and even declared his intention to mould some of them.

Next we determined to produce an animation sequence depicting the gradual metamorphosis between the initial key shapes of the sculptures. This animation was achieved as follows. First, we mapped the abstract coordinates to multimedia coordinates: $x_4 \rightarrow t_1$, $x_5 \rightarrow t_2$. Then, we defined a spiral animation path on the t_1t_2 plane which successively passes through the points labelled 1 through to 21 (see Figure 6). The resulting animation is available at the following Web address: <http://cis.k.hosei.ac.jp/~F-rep/App/ASP/FASP.html>

From Virtual to Physical Sculptures

Having created virtual artefacts, one can produce their physical incarnations using rapid prototyping machinery and the process called “3D printing”. We have been experimenting with a 3D Systems SLA3500 machine⁴². This machine is based on solid-state laser technology (stereolithography), where the laser beam scanning the current horizontal cross section of the object causes solidification of liquid resin. Figure 7a shows an artefact produced with this machine from a virtual shape. The selected shape represents the middle of the metamorphosis triangle. It was defined by setting its x_4 and x_5 coordinates to zero and by using equal weights for all three initial sculptures. The HyperFun model of this virtual shape was polygonized and exported in STL format (a standard polygonal mesh format for rapid prototyping). Then, the 3D Lightyear software, developed by 3D Systems, was used to generate cross-sections of the object. Fig. 7a shows an 8 cm high figurine which took 12 hours to generate. Fig 7b shows the same object produced using stacked and glued paper sheets by a KIRA

Solid Centre machine based on the so-called laminated-object manufacturing process. It is worth observing that after sanding it down the final figurine looks and feels like a wooden sculpture.

Navigation and Sculpting in an Augmented Sculpture Garden

The next stage of development of our computer-mediated-sculpture study was perhaps the most consequential one yet, namely the development of a system that allows the artist to experience real-time interaction with a virtual sculpture using a VR type interface with quite a high level of user immersion.

This system allows the user to navigate interactively through a so-called “FRep Sculpture Garden”⁴³, which is a time-dependent scene composed of multiple objects (represented as volumes). In this virtual environment it is possible for the user to interactively edit, on the fly, the objects populating this environment as well as the processes that metamorphose these objects over time. The initial scene is composed of the three, by now familiar, sculptures that are placed at the vertices of equilateral metamorphosis triangle with its centre serving as a placeholder for the result of the sculpting process. The system allows users to combine two ways of controlling the shape of the sculpture. The first way is concerned with global shape modification and is based on a metamorphosis operation that combines several objects present in the scene. Generation of a metamorphosis shape provides an interesting "initial guess" for an artist and thus stimulates his/her imagination. The second way is intended for local object modification and is done using a metaphor of a carving / melting tool, so the artist handles the shape in a more natural way compared to that of surface tangent controls. This allows for further refining of the generated shape.

Next, let us describe how the user interacts with the model during the sculpting process.

First of all, the user is able to navigate in the initial static scene selecting an appropriate point of view. As the scene needs to be rendered in real time, performance considerations are of paramount importance. To employ off-the-shelf 3D acceleration hardware specialized in polygonal mesh rendering, we convert the initial FRep into an intermediate voxel array representation suitable for editing, which is interactively polygonized using an original algorithm⁴⁴ for the purpose of visualization. We reduce the number of triangles produced using a view-dependent Level-of-Detail (LOD) polygonization technique that utilises a hierarchical representation of the voxelized FRep objects. Figure 8 shows some navigation snapshots when only one LOD is employed, while Figure 9 shows an adaptive level-of-detail applied to one of the models. Even when one LOD was employed we could generate between 25 and 30 frames per second for models voxelized on 64^3 grid. This frame rate is sufficient for interactive navigation. Adaptive LOD reduces the number of triangles generated by a factor of 3 to 4, thus making navigation more responsive (i.e., between 40 to 50 fps). The user can control the detailing of a portion of the scene by positioning the detail refinement tool close to it. This tool is shown as a small green sphere in Figure 9.

Once a point of view is chosen, the user can begin the process of metamorphosis. The point of view can be changed at any time by interrupting the metamorphosis process. The metamorphosis is controlled by the position of the metamorphosis tool. Using this tool, the user can select a point inside the metamorphosis triangle and thus determine the weights of each input sculpture for the intermediate shape shown at the centre of

this triangle. This is an iterative process, which allows the user to arrive at an aesthetically pleasing output shape. Figure 10 shows two snapshots (generated using only one LOD) taken during the metamorphosis process. The metamorphosis process can be interrupted at any stage and the navigation process can be resumed, allowing the user to move towards or around the interpolated output shape (Figure 10b).

Once the desired output shape is arrived at, the user can further refine it using a carving tool which allows the user to either remove material from the shape or to add material to it. During the carving process, the sculpting tool (shown as a blue sphere in Figure 11a) can be immersed into the surface of the shape causing spherical indentations to appear. At any stage, the metamorphosis process can be interrupted and the navigation process can be resumed (Figure 11b). During the carving process, it is even possible to create a new shape ab initio using the sculpting tool. The user can control both the position and the size of the sculpting tool, thus making fairly complex modifications to the initial object. The results of the carving process can be discarded by using the metamorphosis tool to select some other point in the metamorphosis triangle. It is fairly easy to augment this system by simulating a set of carving tools with various shapes that allow the user to perform quite diverse and subtle operations on the resulting shape. Finally, the resulting sculpture can be exported into the HyperFun language file.

The most computationally intensive operation is the metamorphosis as it modifies the whole object thus forcing its re-poligonization. To increase the frame rate during the metamorphosis, partial updates for each frame were implemented. This means that only a portion of the metamorphosis result is updated per frame. In our experiments

we varied the number of updates from $40^3=64,000$ samples per frame to a 1/40th of this, i.e. $40^2=1,600$ sample updates per frame. The relationship between the number of sample updates per frame and the frame rate along with the percentage of processing time spent on various tasks during the navigation / metamorphosis process is shown in Table 1. A similar problem exists for carving operations; we can, however, effectively limit the invalidated mesh region using the bounding box of the carving tool. In our experiments, updates cover sub-volumes varying from 14^3 to 18^3 samples in size, resulting in 2,700 to 5,800 modified samples per frame. As this number of voxel array updates is relatively small, the frame rate was maintained at about 12 to 15 fps, which is close to the navigation frame rate. This means that on the fly modifications of the model are possible.

Another issue is related to the export of the final result. All the editing operations are applied to the intermediate voxel representation, selected to maximise rendering performance. While an FRep is able to incorporate the voxel data, such an approach to the final model generation is not optimal. Moreover, detail resolution becomes fixed and might be insufficient for actual usage of the model in, say, rapid prototyping applications. We can overcome this limitation by directly exporting the object model into the HyperFun language, using the primitives from the language library and CSG operations to approximate the results of the editing operations applied to source objects. Thus, the carving operations are represented in the final model as a subtraction or a union operation with the object bounded by a convolution surface controlled by a carving path and the diameter of the carving tool. We describe the carving results as a set of points resulting from trajectory sampling and the corresponding carver diameter.

For the metamorphosis operations, it is sufficient to export the weights of the original sculptures in the final result.

Our initial experience with this system shows that it is easy to master, even by children. The system is efficient enough to be able to maintain interactivity with all operations even on an ordinary personal computer with an average graphics card. A recorded session of interactive metamorphosis and carving can be downloaded from: <http://cis.k.hosei.ac.jp/~F-rep/App/ASP/FASP.html>

The Augmented Sculpture Installation Project

In this section we outline a proposal for an interactive art installation (more details can be found in Adzhiev et al⁴⁵). A rough sketch of the basic installation is shown in Figure 12. Its title reflects a form of sculpture which is both physical and virtual and where the virtual world is overlaid on the physical world. The physical aspects of this installation will include the physical sculptures, and the space in which this installation is exhibited. The virtual aspects of this installation will include a series of computer-generated images and animations depicting the sculpting process (as described in the previous section) which will be displayed and projected on various monitors and surfaces of the exhibit.

The physical sculptures of our resident sculptor will constitute the key-shapes that will give rise to the virtual artefacts produced by the interactive sculpting sessions. It is intended that the viewers of the exhibit are active participants in the sculpting process. Viewers will be encouraged to interact with the exhibit and to produce their own virtual sculptures in real-time. Some computer-generated views and animations will be

dependent on the position and orientation of the viewer inside the exhibit. Thus, the viewer will be able to experience and affect the virtual part of the exhibit.

The sketch in Figure 13 shows a triangular pyramid at the vertices of which are situated four physical sculptures that will act as the key-shapes of the interpolation process. In the middle of the triangular base of the pyramid lies a fifth sculpture that is the result of the interpolation of the three sculptures at the corners of the base. One also may use a rapid prototyping machine to manufacture a physical version of the created virtual sculpture.

Conclusion

In conclusion we would like to make the following observations.

The functional representation of shapes that we have employed presents the system user with the opportunity to envision, discover and create novel sculptural forms without imposing many restrictions.

Further we observe that, this type of modelling, combined with evolutionary algorithms and virtual reality interfaces, point to a wealth of research possibilities. The full cycle of the artistic endeavour has been tested in our work: starting from real sculptures, modelling them, obtaining new virtual sculpture shapes through metamorphosis and virtual sculpting, interactively modifying these virtual shapes and generating new physical sculptures from these virtual shapes.

Our initial, but as yet incomplete, study of the creative cycle from physical artefact to computer model and back to real sculpture, through the use of computer-aided manufacturing or rapid prototyping methods, constitutes the basis for a novel sculpting

technique. Augmenting this form of sculpture with animation leads to a new art form that enables the experimental discovery and realisation of computer inspired and mediated physical sculptures.

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Number of sample updates per frame	64000	32000	16000	8000	6400	3200	1600
Frame rate (frames per second)	1.37	2.56	4.56	7.41	8.45	11.65	14.04
Number of visible cells	21613						
Number of rendered triangles	23496						
Total processing time as a % of frame duration	95.2	92.7	84.4	74.8	71.0	59.4	52.0
Metamorphosis handling time as a % of frame duration	92.9	86.5	75.3	61.5	55.8	39.6	26.7
Visibility checking for non-empty cells time as a % of frame duration	2.3	4.5	7.3	11.9	13.5	18.6	22.5

Table 1. The relationship between the frame rate and processing costs, and the rate of sample updates per second.



a

b

c

Figure 1. The physical sculptures: (a) “Naked”, (b) “Gymnast”, (c) “Walking Androgynous”.

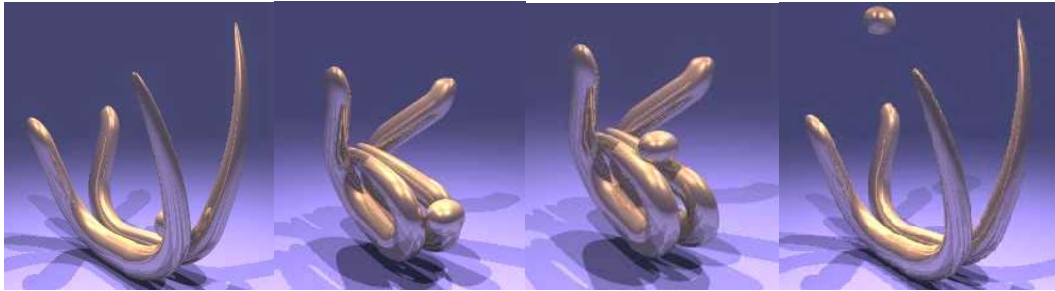


a

b

c

Figure 2. The Geometric models: (a) “Naked”, (b) “Gymnast”, (c) “Walking Androgynous”.



a

b

c

d

Figure 3 (a-d). Four frames of the Gymnast animation.

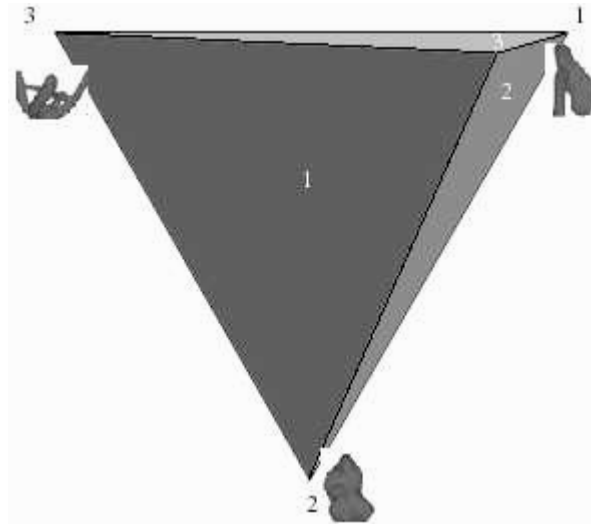


Figure 4: The metamorphosis computational scheme

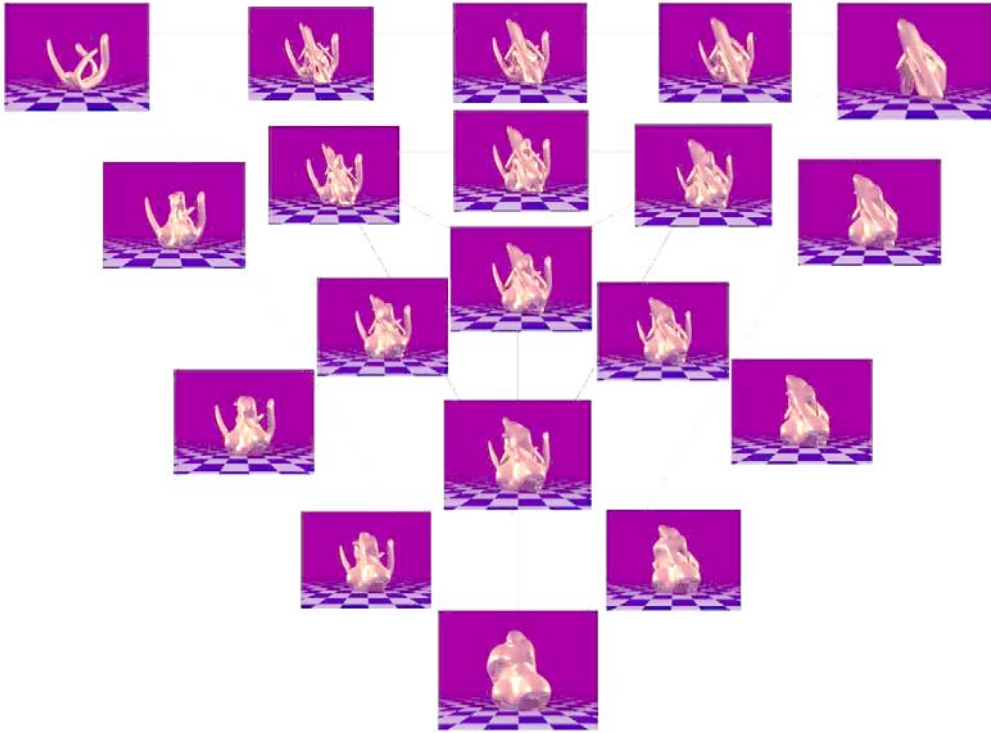


Figure 5. The metamorphosis “triangle spreadsheet”

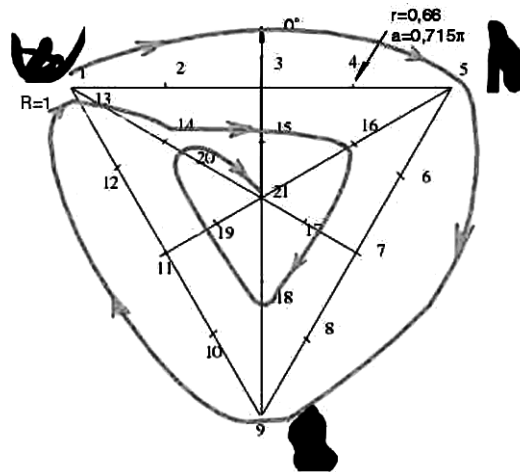


Figure 6. An animation path on the t_1t_2 plane

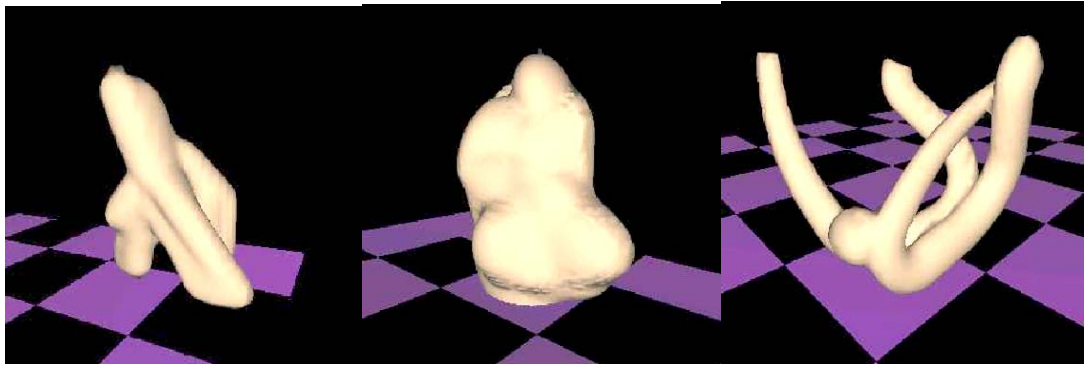


a



b

Figure 7. Rapid prototyping of a physical sculpture generated from
a) resin using the SLA3500 machine; b) paper using the KIRA Solid Centre system.



a

b

c

Figure 8 (a-c). Several snapshots of the scene made during navigation. Only base level of detail is used.

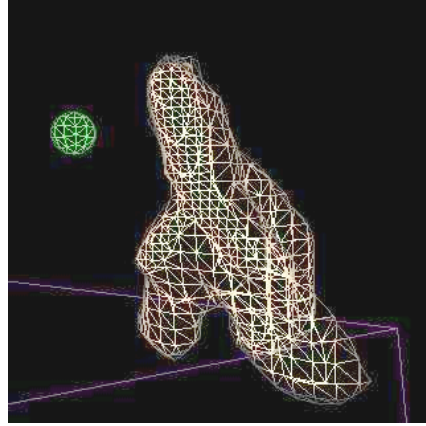
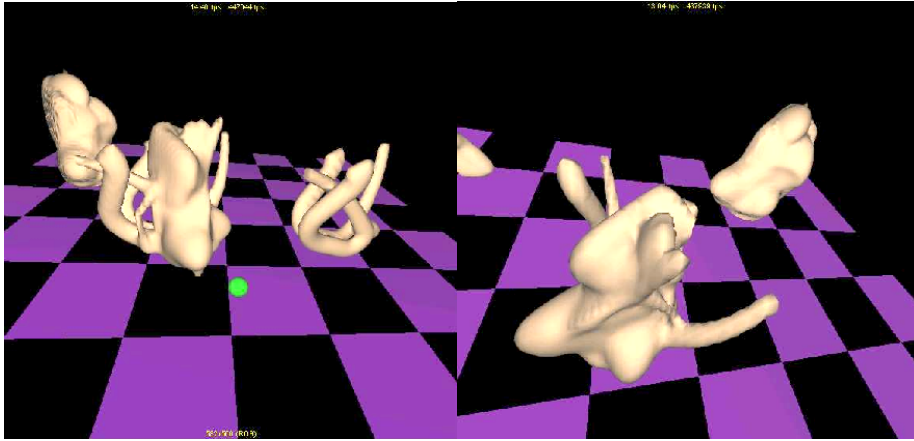


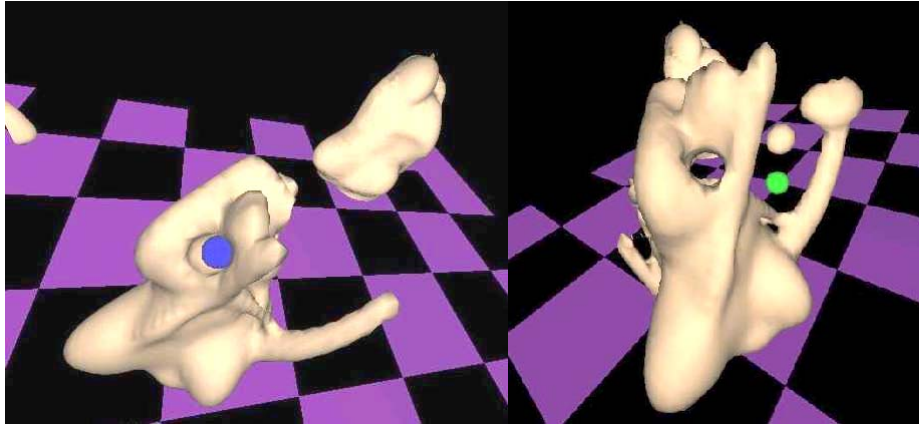
Figure 9. Demonstration of the adaptive level-of-detail applied during isosurface polygonization. The green sphere represents the detail refinement tool.



a

b

Figure 10 (a-b). Two snapshots taken during metamorphosis of the source sculptures.



a

b

Figure 11 (a-b). Two steps during carving on the metamorphosis result from Figure 9b.

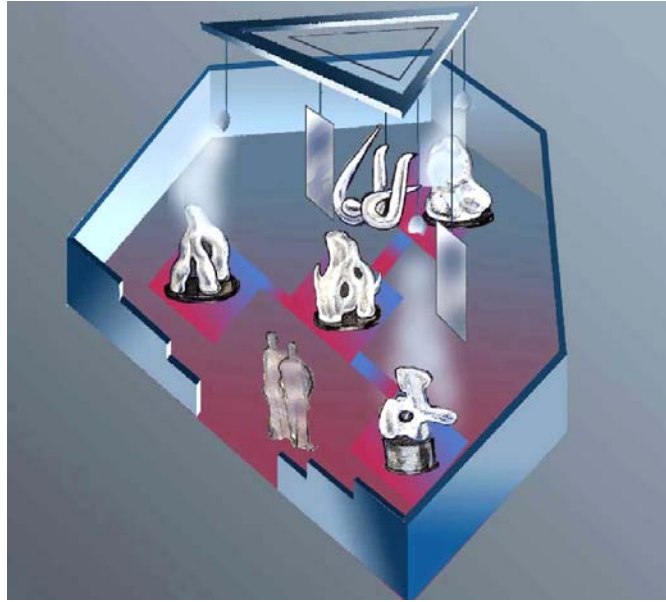


Figure 12. A rough sketch of the Augmented Sculpture installation.